Probabilistic Value of Life vs. Deterministic Value of Time

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Abstract

This paper re-examines the standard probabilistic willingness-to-pay value of life model of Jones-Lee and others. A consumer decides on allocating his income between consumption, life insurance, and precaution — which affects his probability of survival in the one future period. One innovation of the paper is to assume a simple power utility function, and to solve the probabilistic value of life model analytically.

I show that the standard probabilistic value of life measure has some strikingly bad features, including a strong negative connection with altruism.

I then discuss an alternative to the probabilistic value of life model. The alternative is a certainty model, in which a consumer decides how much to spend to increase the certain length of his life. I argue that the certainty buying time model has many advantages over the probabilistic value of life model, that it avoids several problems and paradoxes associated with the probabilistic model, and that it is more consistent with our intuition.

Keywords

Value of life, value of time, willingness-to-pay, Broome paradox, death

Classification Code

D11, D61, D6, D91

I. **Introduction**

What is the money value of a human life?

This interesting question has a more-or-less standard answer in the legal sphere, where the usual approach to valuing a life is the *human capital* approach. (For surveys, see, e.g., Keeton (1984), Speiser (1975), Brookshire (1987).) The deceased is viewed primarily as a money-making machine. The value of his life is mainly given by lifetime income or earnings, possibly net of expenses needed to maintain the machine, possibly discounted to present value, and possibly augmented by imputed income for non-market services.

However, this is not the standard approach to value of life taken by many economists, and it is no longer the standard approach of government policy makers. The human capital approach is rejected because, while it measures earning capacity, it does not measure how much the deceased valued his own life. To measure how much people actually value their own lives, it is said, we should see how much they are willing to pay to avoid fatal risks. This is the *probabilistic willingness-to-pay* approach, and it is now the standard economic theory of value of life. (See Jones-Lee (1974), Jones-Lee (1976), Mishan (1971), Mishan (1982a), Schelling (1968), Schelling (1987), Viscusi (1992), and others.)

The probabilistic willingness-to-pay approach essentially proceeds by posing the following question: Suppose a consumer has an opportunity to make his life safer, but at a cost. Assume he can reduce his probability of dying by ϵ if he buys some product, service, or government program. What is the maximum, say C, he would pay for the extra survival probability? The consumer decides on some C for the given ϵ. Then his valuation of the survival probability increment ϵ is C. The probabilistic willingness-to-pay value of his *whole life* is said to be C/ϵ. When there are n identical individuals being asked the same sort of question, they reveal that they would pay a total of $n \cdot C$ to prevent $n \cdot \epsilon$ expected deaths, which gives C/ϵ per expected death prevented.

(Alternatively, the consumer may be faced with an increase in the probability of dying ϵ, and he may be asked for the minimum C that he would accept, in exchange for the greater risk of death. The value of life is again C/ϵ, and this is, strictly speaking, a *probabilistic willingness-to-accept* measure. In what follows I will lump together the willingness-to-pay approach and the willingness-to-accept approach, and call both probabilistic willingness-to-pay measures.)

There have been many studies which attempt to estimate C/ϵ. Good surveys can be found in Fisher et al. (1989); Jones-Lee (1989), Chapter 2; Viscusi (1992), Chapters 3 & 4; and Viscusi (1993) among others. This is not an empirical paper, and it is not my intention to criticize the empirical work. However, I will make a few observations: First, many of the empirical studies find values of life that exceed the human capital measure by roughly an order of magnitude. Second, the studies get wildly differing estimates for the value of life, and many absurd results (for example, that union workers value their lives much more highly than non-union workers.)

Third, the empiricists seem to have overlooked many common behaviors that appear to be *risk-seeking*. Behaviors like skiing, whitewater canoeing and kayaking, skydiving, automobile and motorcycle racing, bungee jumping, rock climbing, mountaineering, and spelunking all come to mind, not to mention warfare. For instance, skiers and alpine climbers at Mont Blanc in France suffer more than two hundred fatalities per year. Climbing to the summit of Mt. Everest appears to carry a risk of death of approximately 20 percent, and yet, according to recent news stories, some people pay $60,000 to guides and expedition outfitters for the opportunity. Perhaps if the effort were expended, empirical evidence would be amassed that "proves" people are willing to pay to *increase* risk of death. It appears to me, for instance, that a study of people who climb Mt. Everest would establish a value of life of around $C/\epsilon = -\$300,000$.

Fourth, in the United States the probabilistic willingness-to-pay approach has started to leak into courtrooms under the name "hedonic damages." Here willingness-to-pay is used to attach a dollar value to the life of an individual already dead, with the damages going to his estate or survivors. Several authors have observed that this is a misuse of the method (e.g., Dickens (1990), and Viscusi (1992)).

The probabilistic willingness-to-pay approach has been attacked on philosophical and social-choice-theory grounds. (See particularly Broome (1978, 1982, 1985), Ulph (1982), Blackorby and Donaldson (1986), and Blackorby, Bossert and Donaldson (1993).) One interesting philosophical attack is what I would call the Broome paradox (Broome, 1978): Suppose the government plans a project that will cost some money and also create a risk to some lives (for instance, a long tunnel). Assume there are n identical people whose lives are put at risk, and the project creates an additional risk of death for these people of ϵ. Assume that they would require at least C each to accept that additional risk of death. (Here C is a willingness-to-accept number, strictly speaking.) Then by reasoning like what's above the probabilistic willingness-to-pay value of life is C/ϵ. The expected number of lives that the project will cost is $n\epsilon$, and the "life cost" of the project is nC. This figure might be acceptable to a government planner, who might find it compares favorably to the project's net social benefit, aside from the cost of lost lives. But, argues Broome, what if we knew *which* $n\epsilon$ *were to die*, what if we know their names? And what if we approached the $n\epsilon$ named people, and asked what amount of money they would require to compensate for the certainty of death? They would each answer with an extremely high figure (perhaps infinite), and now the life cost figure would far exceed the project's net social benefit. The project would fail a cost-benefit test by a spectacular margin. So Broome argues that the willingness-to-pay approach is philosophically unacceptable. It leads to conclusions based on *ex ante* information which almost surely would be rejected with *ex post* information.

Like the papers cited above, the purpose of this paper is to attack the theoretical foundation of probabilistic willingness-to-pay value of life measure. In order to do this I will first lay out a simple two-period, two-state probabilistic value of life model, similar to what has been used traditionally. (See, e.g., Jones-Lee (1974 and 1976) among others.) The innovation here is to make a few assumptions that allow analytic solutions to the model. Using the solved model, I define the standard probabilistic willingness-to-pay value of life measure.

The model used here incorporates insurance (as does, e.g., Bailey (1978), Bergstrom (1982), Conley (1976), Cook & Graham (1977), Dehez & Dreze (1982), Jones-Lee (1980)) as well as bequest-motives (as does, e.g., Jones-Lee (1974), and many others). It incorporates precaution as a choice variable, which is somewhat unusual.

The model has two parameters that are especially important. The first is a measure of the individual's concern for his dependents (or his beneficiaries); this is the *bequest parameter*, or what I call *altruism.* The second is a measure of the individual's *fear of death.* I show that for the standard probabilistic willingness-to-pay measure of the value of life, value of life will depend crucially on these two parameters. The probabilistic willingness-to-pay value of life measure is strongly negatively related to altruism, and strongly positively related to fear of death.

After I present the standard probabilistic value of life model, I lay out a simple version of what I believe is a better alternative: a deterministic *value of time* model. In the *buying time* model, the individual does not decide how much to spend to reduce the probability of death (a state of which he has no direct experience), rather, he decides how much to spend to increase the (certain) length of his life. The structure of the deterministic buying time model is somewhat similar to the structure of the probabilistic value of life model; however, the model has implications that are plausible when the probabilistic model's implications are implausible. I will argue that probabilistic value of life model leads to untenable conclusions, whereas the deterministic buying time model doesn't.

II. **The Probabilistic Willingness-to-Pay Value of Life Model**

This is a two-period model, with a "before" and an "after", or *ex ante* and *ex post.*

A two-period model is clearly less realistic than a multi-period model or a continuous time model, but the lack of realism is compensated for by tractability. "Before" is called time zero; "after" is called time one.

At time zero, an individual is contemplating his state at time one. In that future period, he will be either alive or dead. If alive, his utility (as contemplated from time zero) will depend on the amount of money he has to spend. If dead, his utility (as contemplated from time zero) will depend on the amount of money in his estate, and on a factor that represents his "fear of death." This is, of course, utility as seen *ex ante,* for if he is dead *ex post,* he then has no utility (as far as we know).

Our individual is endowed with a given sum of money at time zero. He may use it in three ways: (1) He may hold it to spend on consumption at time one, or to bequeath, as the case may be. The money he uses this way is called *consumption.* (2) He may spend it at time zero on *precaution.* By spending it on precaution, he increases the probability that he will be alive at time one. (3) He may spend it at time zero one on *life insurance.* By spending it on life insurance, he increases his bequest by the face value of whatever life insurance policy he buys.

I assume that the individual has an expected utility function. His maximization problem is to choose the sums of money used in the three ways detailed above, so as to achieve the highest *ex ante* expected utility.

For this model I use the following notation:

x	$=$	amount of money to be spent on consumption (or bequeathed if dead) at time one
y	$=$	money spent on precaution at time zero
z	$=$	money spent to purchase life insurance at time zero
$\overline{x}$	$=$	$x + y + z =$ cash endowment at time zero
$p(y)$	$=$	probability of life at time one
$1 - p(y)$	$=$	probability of death at time one
$V(y, z)$	$=$	face value of life insurance policy
$v(y)$	$=$	price of insurance, per dollar of face value
$f(x)$	$=$	ex ante utility contingent on life
$g(x + V)$	$=$	ex ante utility contingent on death
A	$=$	bequest parameter, or "altruism"
K	$=$	"fear of death" parameter

As in the standard model, I assume the subject chooses x, y and z so as to maximize expected utility

$$Eu = p(y)f(x) + (1 - p(y))g(x + V(y, z)),$$

subject to the constraints

$$\overline{x} = x + y + z, \quad x, y \geq 0, \quad \text{and} \quad x + V \geq 0.$$

I don't require that z be nonnegative because it is analytically simpler to allow individuals to buy negative quantities of life insurance (i.e., rather like annuities, translated into the structure of this model). I do require that consumption, precaution, and the estate $x + V$ be non-negative.

To solve the model, I make the following special assumptions:

<u>Assumption 1</u> (Actuarially fair life insurance).

$$V = \frac{z}{1 - p(y)}.$$

That is, the premium for the life insurance policy z equals the expected payout, $V \cdot [1 - p(y)]$. To put it slightly differently, the price for a dollar's worth of insurance is $v = 1/(1 - p(y))$.

There are two possible alternative assumptions about insurance companies' pricing policies: They may be able to observe each individual's degree of precaution y, and price accordingly (offering each individual a schedule of rates, depending on y). Or they may not (offering each individual one price, $1/(1-p)$). The latter assumption produces a slightly simpler model, and I will follow it here. It implies that, when choosing x, y, and z, the utility-maximizing individual assumes $\frac{\partial V}{\partial y} = 0$, or, to say the same thing, he views the price v of a unit of insurance as a constant, rather than as a function of y.

Assumption 2 (Utility if dead).

$$g(x+V) = Af(x+V) - K,$$

where A and K are non-negative constants. That is, the utility from the dead state is comprised of (a) a scaled up (or down) version of utility if alive, minus (b) a *fear of death parameter* K. The parameter A is a measure of the subject's concern for his bequest. If he is very concerned about the welfare of his heirs, A might be larger than 1. If he places the same weight on the financial well-being of his heirs as he places on his own financial well-being, he might have $A = 1$. If he has no dependents and no interest in a bequest, $A = 0$. A will be called the *altruism* or the *bequest parameter.*

The fear of death parameter reflects the disutility of death per se, viewed ex ante. If the subject doesn't care whether he lives or dies (aside from the issue of a bequest), $K = 0$. If he does want to avoid death, $K > 0$.

Assumption 3. (Power utility function).

$$f(x) = x^{\alpha}, \quad \text{for some } 0 < \alpha < 1.$$

The power utility function provides some degree of generality, but still permits analytic solutions of the model. Note that $1-\alpha$ is Arrow's relative risk-aversion measure. (An alternative although slightly less general approach would be to let $f(x) = ln\ x$. This would produce a model quite similar to what is developed below.)

When assumption 3 is *not* made, I assume that the function $f(\cdot)$ satisfies the following regularity conditions: For $x \geq 0$, $f(\cdot)$ is non-negative, differentiable, strictly increasing and strictly concave.

Assumption 4. (Survival probability).

The function $p(\cdot)$ satisfies the following regularity conditions: For $y \geq 0$, $p(\cdot)$ is positive, differentiable, strictly increasing and strictly concave.

III. **Solving the Probabilistic Willingness-to-Pay Value of Life Model**

In general terms, our individual wants to maximize expected utility subject to certain constraints. Using only assumptions 1 and 2, the problem is to maximize:

$$E(u) = p(y)f(x) + (1 - p(y))[Af(x + vz) - K], \text{ subject to}$$

$$x + y + z = \overline{x};\ x, y \geq 0, \quad \text{and} \quad x + vz \geq 0.$$

First-order conditions for maximization lead to the following equations (assuming an interior maximum, and incorporating assumptions 1 and 2):

(1) $$f'(x) = Af'(x + vz).$$

(2) $$p'\ [f(x) - (Af(x + vz) - K)] = f'(x) = \lambda.$$

Here the primes denote derivatives, and λ is the Lagrange multiplier associated with the budget constraint.

Now consider equation (2). The intuitive interpretation of p' is crucial. This is the marginal increase in survival probability resulting from a marginal increase in precaution expenditure y, for the utility maximizing individual who is choosing precaution, along with x and z. The inverse of p' is the marginal increase in precaution expenditure corresponding to a marginal increase in survival probability.

That is, $1/p'$ *represents precisely the utility maximizing individual's marginally measured willingness-to-pay value of life.* It is exactly analogous to the C/ϵ measure discussed above. (See also Bailey (1978), and others.) Therefore, $1/p'$ *is the standard probabilistic measure of the value of life, and in what follow, I let* VOL *stand for this measure.*

Hence, from (2)

(3) $$VOL = \frac{1}{p'} = \frac{1}{f'(x)}\ [f(x) - (Af(x+vz) - K)].$$

The interpretation of equation (3) is simple and obvious:

(4) $$VOL = \frac{\text{Utility advantage from being alive over being dead}}{\text{Marginal utility of money spent on consumption}}.$$

Now I use assumption 3, which, when combined with equation (1) above, gives:

(5) $$x + vz = A^{\frac{1}{1-\alpha}}\ x,\ \text{or}\ V = vz = (A^{\frac{1}{1-\alpha}} - 1)\ x.$$

It follows that the non-negative estate constraint $(x + vz \geq 0)$ is automatically satisfied as long as $x \geq 0$, because of the $A \geq 0$ assumption.

Next, substituting for $f(x)$ and for $x + vz$ in equation (3) gives:

(6) $$VOL = \frac{1}{p'} = \frac{1}{\alpha x^{\alpha-1}}\ [x^{\alpha}(1 - A^{\frac{1}{1-\alpha}}) + K].$$

In the event that $A = 0$ (no bequest motive),

$$(7) \qquad VOL = \frac{1}{\alpha x^{\alpha-1}} \left[x^\alpha + K\right], \text{ and } x + vz = 0.$$

In the event that $A = 0$ (no bequest motive), *and* no insurance market exists (so that z is constrained to be zero), first-order maximization conditions lead directly to

$$(8) \qquad VOL = \frac{1}{p} \frac{1}{\alpha x^{\alpha-1}} \left[x^\alpha + K\right].$$

Note the difference between equation (8), which makes VOL inversely related to survival probability, and equations (3) and (7), which do not. That is, *the inclusion of insurance (in this case negative insurance) cuts the link between survival probability and the value of life.* (This is a general result, for $A \geq 0$.)

Finally, in the event that $A = 1$, equation (6) reduces to a very simple and attractive form:

$$(9) \qquad VOL = \frac{K}{\alpha x^{\alpha-1}}.$$

In order to define a John Broome-style alternative to the marginal VOL measure, I propose the following question: Suppose a subject has some bequest motive, so that $A > 0$. Suppose he has chosen utility maximizing x, y and z. Confront him and ask the following: What addition to your estate $\tilde{X}$ would you require, so as to be indifferent (ex ante), between the live state and the dead state? I'll call the answer to this question the *life-death indifference* measure of the value of life. In general, it is the solution to

$$(10) \qquad f(x) = Af(x + vz + \tilde{X}) - K.$$

Equation (10) has only incorporated assumption 2. Using assumptions 1, 2 and 3 together gives:

(11) $$\tilde{X} = A^{-\frac{1}{\alpha}} \, (x^\alpha + K)^{\frac{1}{\alpha}} - A^{\frac{1}{1-\alpha}} \, x.$$

It would be helpful at this point to get an idea of what are appropriate magnitudes for the parameters of the model. The current average annual wage in the United States is in the neighborhood of \$25,000; so let us assume that $x = \$25,000$. The parameter α is constrained to be between 0 and 1; let us assume the simple midpoint value of $\alpha = 0.5$. By equation (5), $A^2 + 1$ should equal the ratio of insurance face value vz to consumption x; a plausible ratio would be 3; which gives $A = 2$ as a plausible value. Substituting these values into equation (6) gives a simple linear relationship between VOL and K.

At this point we are at a temporary impasse because there is no good independent way to guestimate the fear of death parameter K. If you believe empirical studies of probabilistic willingness-to-pay value of life (and I really do not), then VOL should be in the range of \$1 to \$10 million. Let us therefore assume $VOL =$ \$5.0 million. It follows that $K = 16,286$.

Substituting into equation (10) then gives $\tilde{X} = \$67,500,000$. This subject acts as if his probabilistic value of life is \$5.0 million, but he would require an extra \$67.5 million added to his bequest to be indifferent between the live and dead states. All this while his chosen level of consumption is \$25,000.

IV. **How Does VOL Depend on K and A?**

Two obvious questions come to mind about VOL, the economists' favorite measure of value of life. First, how does VOL depend on A the altruism parameter? And second, how does VOL depend on K, the fear of death parameter?

In general, with a fixed initial endowment $\overline{x}$, an individual will choose x, y, and z based on the parameters A, K, and α. Normally, in order to calculate the response of VOL to changes in K (or A), one would hold $\overline{x}$ fixed, and let x, y and z vary as the parameter K (or A) varies. However, it is much easier in this model to hold x fixed, and let $\overline{x}, y$ and z vary as A (or K) varies. Therefore, this is what I shall do for comparative statics. In the equations below, the derivatives hold x and the obvious parameters constant, and allow $\overline{x}, y$ and z to vary. I will also assume for this purposes of this discussion that $x > 0$.

First, how does the VOL measure vary with the altruism parameter A?

We first consider the general case, without assumption 3, the assumption that $f(x) = x^{\alpha}$. From equation (3) we get

$$(12) \qquad \frac{dVOL}{dA}|_x \text{ constant} = -\frac{1}{f'(x)} \; [f(x+vz) + Af'(x+vz)\frac{d}{dA}(x+vz)]$$

Using equation (1) it is easy to show that

$$\frac{d(x+vz)}{dA}|_x \text{ constant} = -\frac{f'(x+vz)}{Af''(x+vz)}.$$

Based on the regularity assumptions for $f(\cdot)$ (which imply that $f(x) > 0$ for $x > 0$; and $f'(x) > 0$ and $f''(x) < 0$, when $x \geq 0$), we can conclude that:

$$\frac{dVOL}{dA} |_x \text{ constant} \leq 0,$$

with the strict inequality holding if $x + vz > 0$.

With assumption 3, we can use equation (6) for VOL, and get the following:

$$\frac{dVOL}{dA}|_x \text{ constant} = -x \; \left(\frac{\alpha}{1-\alpha}\right) \; A^{\frac{\alpha}{1-\alpha}} \leq 0,$$

with the strict inequality holding if $A > 0$. In short, an altruism rises, the probabilistic willingness-to-pay value of life falls.

But it is worse than that. Not only does VOL fall as A rises, but it falls rather rapidly to zero. For example, with parameters somewhat similar to what was assumed at the end of the last section $(A = 2, \alpha = 0.5, K = 3637)$, and with the same assumed level for utility maximizing consumption $(x = \$25,000)$, we would have $VOL = \$1,000,000$. Maintaining all of these values except A, and increasing A to 3.0, for instance, would cause VOL to drop to $750,000; increasing A to 4.0 would cause VOL to drop to $400,000; and VOL would reach $0 at around $A = 4.90$. (To see what might be a plausible value for A in this case, recall from equation (5) that the ratio of face value of insurance policy to consumption should equal $A^{\frac{1}{1-\alpha}} - 1$. With $A = 4.9$, $\alpha = 0.5$, and $x = \$25,000$, an individual would have a $575,000 insurance policy.)

In my view, this strongly negative relationship between A and VOL suggests that basing social policies on VOL might be unwise.

Second, how does VOL vary with the fear of death parameter K?

In the general case (without assumption 3), we differentiate equation (3) to get

$$\frac{dVOL}{dK}|_x \text{ constant} = \frac{1}{f'(x)} > 0. \tag{13}$$

With assumption 3, we differentiate equation (6) to get

$$\frac{dVOL}{dK}|_x \text{ constant} = \frac{1}{\alpha x^{\alpha-1}} > 0. \tag{14}$$

In short, as fear of death rises, the probabilistic willingness-to-pay value of life measure rises.

This positive relationship between K and VOL is of course to be expected. After all, willingness-to-pay models of value of life get their impetus from the natural distaste for the dead state. But I find something philosophically unattractive about a theory that leans so much on fear of an unknown and unknowable outcome. There is *ex ante* disutility from the dead state but *ex post*, there is no known disutility or utility. To paraphrase the ancient Greek philosopher Epicurus, we fear death awfully when we are alive, but when we fear it we *are alive*, and when we are dead we will not fear it any more.

When I discuss the standard probabilistic willingness-to-pay value of life model with undergraduates, and I ask them to "assume a utility if dead function $g(x + V)$," they are mystified. They believe that either (i) there is no *ex post* utility if dead, or (ii) ex post utility if dead is unknowable, or (iii) ex post utility if dead is somehow defined by religion, and might be blissful (if heaven), or awful (if hell), or otherwise (if reincarnated). *Ex ante* utility if dead is one remove from *ex post* utility, and therefore even more bizarre: What is it? How would it be weighed or measured? And the fear of death parameter K is, of course, similarly ill-defined. It is certainly the case that most people fear dying, but quantifying either the an ex ante utility if dead function $g(\cdot)$, or a fear of death parameter K, should require (at the least) complete and transitive preferences over alternate alive and dead states. There is certainly no evidence that such complete and transitive preferences exist.

In short, basing a theory of utility maximization on a utility function half of which makes no sense to most people, seems to me to be ill-advised.

V. The Better Approach – Buying Time – The Deterministic Value of Time Model

In my opinion, the willingness-to-pay approach should be put on a firmer foundation. An individual should not be asked to hypothetically evaluate death, which he does not know. An individual *should* be asked to evaluate additional years of life, which he *does* know. The model should not be about buying extra probability of survival, it should be about buying extra years of life. The unknowable dead state should be set aside, and the choice should be about additional units of a known live state.

The simplest way to do this is to put aside the probabilistic model, and just construct a model of buying additional time in a certainty context. (Buying time is an uncertainty context is also a reasonable metaphor, but it is much more complex, and a great deal can be learned from the simpler certainty model.)

The model below is based in part on ideas in Jones-Lee (1976), Kenyon and McCandless (1984), and Moore and Viscusi (1988); and most particularly on certainty models in Ehrlich and Chuma (1990), and Feldman (1997). The subject decides on consumption, life extension and bequest; and life extension expenditures, instead of increasing survival probability as in the probabilistic model, increase the length of life. So the subject reveals his willingness-to-pay for years of life.

At this point, I will replace the notation and definitions of section II. I will use the following notation and definitions:

Lu	=	lifetime utility
x	=	instantaneous rate of spending on consumption, assumed constant over the lifetime
y	=	money spent by the subject on life extension, assumed spent in one lump sum at time zero, when the plan is made
z	=	bequest, assumed set aside in one lump sum at time of zero, when the plan is made.
$\overline{x}$	=	$x + y + z =$ cash endowment
A	=	the bequest or altruism parameter. $A = 0$ indicates the individual has no interest in the size of his bequest, whereas $A > 0$ indicates the individual gets utility from his bequest.
T	=	length of life.
$u(x)$	=	instantaneous utility function.

Assume for simplicity that the instantaneous utility function $u(\cdot)$ is constant over the lifetime, and has the usual nice properties; assume further that the individual's discount rate and the market interest rate are zero. (It is possible to assume a positive individual discount rate and a positive market interest rate, but this produces a significantly more complex model, the conclusions of which are generally similar to what is shown below.) To make the model easy to solve, suppose utility from the bequest z is $Au(z)$. (This is parallel to assumption 2 in section II.) Note that with no uncertainty there is no life insurance.

Now the subject chooses $x, y,$ and z so as to maximize certain lifetime utility:

$$Lu = \int_0^T u(x)dt + Au(z) = Tu(x) + Au(z). \tag{15}$$

There are two constraints. First is a more-or-less standard budget constraint. Consumption takes place at a rate x over a lifetime of length T. Life extension expenditure y and bequest z are made at one moment. There is no discounting in the budget.

Therefore, the budget constraint is:

$$\overline{x} = xT + y + z. \tag{16}$$

The second constraint is crucial. In the probabilistic model, the more that is spent on precaution, the greater is the survival probability in the next period. In this deterministic model, the more that is spent on life extension, the longer is the lifespan. That is:

$$T = \ell(y), \tag{17}$$

where $\ell(\cdot)$ is a positive and increasing function satisfying the same regularity assumptions as $p(\cdot)$ in the probabilistic model. (See assumption 4.)

Recall that in the probabilistic model, the standard probabilistic willingness-to-pay value of life is $VOL = \frac{1}{p'}$.

The analogous concept is the deterministic model is *value of time*, which I will abbreviate VOT. *It is the marginal increase in life extension expenditure per extra unit of length of life, for the utility-maximizing individual who is choosing* x, y *and* z. It is defined as:

$$VOT = \frac{1}{\ell'} = \frac{\text{Extra expenditure on life extension}}{\text{Extra years of life}}. \tag{18}$$

It is easy to show that first-order conditions for an interior maximum for this model include the following equations:

$$u'(x) = Au'(z), \quad \text{and} \tag{19}$$

$$VOT = \frac{1}{\ell'} = \frac{u(x)}{u'(x)} - x. \tag{20}$$

Note the similarity between equation (19) and equation (1) of the probabilistic model, and note the simple formula for the value of time in the certainty model.

Now we can make an assumption similar to assumption 3 of the probabilistic model.

Assumption 5 (Power function instantaneous utility).

$$u(x) = x^{\alpha}, \quad \text{for some} \quad 0 < \alpha < 1.$$

When assumption 5 is incorporated in equation (20), the result is:

$$(21) \qquad VOT = \left(\frac{1-\alpha}{\alpha}\right) x.$$

If it so happens that $\alpha = 0.5$ (as in the examples in sections III and IV), then the willingness-to-pay value of one extra year of life is exactly equal to the annual rate of expenditure on consumption!

At this point let me contrast the two approaches to value of life, contained in equation (6), for the probabilistic model, and equation (21), for the buying time model. I rewrite equation (6) below for the purposes of this comparison:

$$(6) \qquad VOL = \frac{1}{\alpha}\, x \,(1 - A^{\frac{1}{1-\alpha}}) + \frac{1}{\alpha}\, x^{1-\alpha}\, K.$$

The first thing to observe is that the derivation of the *buying probability* model depends on an unintuitive philosophical elephant in the living room, the assumed ex ante utility if dead. What is it? Where does it come from? Whether in the abstract form $g(x+V)$, or the more concrete form involving A and K, there is an overwhelming feeling of unreality in the notion of utility if dead. In the *buying time* model there is no need to resort to ex ante utility if dead.

Second, the buying probability measure VOL is, for a given level of consumption x, strongly negatively related to altruism A and positively related to fear of death K. Since altruism is often considered a virtue, and fear is often considered a vice, these characteristics of VOL might be viewed with suspicion.

On the other hand, the buying time measure VOT is utterly unconnected to any notion of fear of death. Moreover, for a given rate of consumption, it is not dependent on altruism since A does not appear in equation (20) or (21). (Note, however, that with a fixed cash endowment $\overline{x}$, comparative statics analysis does establish a weak negative relation between A and VOT. See Feldman (1997).)

Third, according to empirical studies, the VOL measure is roughly an order of magnitude greater than the human capital value of life. The parameter closest to human capital in both the models of this paper is $\overline{x}$. However, this is a one-period endowment in the buying probability model, a lifetime endowment in the buying time model. The x variable is both models is consumption per period, e.g., per year, and is reasonably close to human capital per unit time. To translate VOL and $\tilde{X}$ to per-unit time equivalents, they should probably be divided by a plausible lifetime number of years, say 50. (This is considerably less than a typical life expectancy at birth, but considerably more than a typical worklife expectancy.) Applying these observations to the numerical example at the end of section III suggests human capital of (very roughly) \$25,000 per period, VOL at around (very roughly) \$100,000 per period, and $\tilde{X}$ – the Broome-like measure – of (very roughly) \$1,350,000 per period. The probabilistic measures are considerably greater than the human capital measure. But the buying time measure VOT is exactly equal to per period consumption when $\alpha = 0.5$, by equation (21). So buying time and human capital measures of the value of life are in agreement. The value of life according to the buying time theory is back in the ballpark of legal tradition.

Fourth and finally, the buying probability model creates inevitable logical paradoxes, and other problems, that the buying time model avoids. These paradoxes and problems of the buying probability model include the following. (A) The Broome paradox (Broome 1985) and the general possibility of inconsistency between ex ante welfare measures and ex post welfare measures. B) The host of problems inherent in performing social welfare comparisons of situations with differing populations. (See, e.g., Blackorby and Donaldson (1986), and Blackorby, Bossert and Donaldson (1993).) (C) The problem of wildly inconsistent empirical studies which purport to measure the same thing, called "value of life."

The buying time model has no ex ante and ex post to contradict one another. The buying time model has no implicit comparisons of alternate populations. One person is choosing a characteristic of his life, its length. Finally, the buying time model is not based on the philosophically bizarre notion of utility if dead. Rather, it is about life, whose utility we can reasonably judge.

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