Lecture 3: rigid body dynamics

- *kinematics example: rolling no slip*
- *rotational equation of motion*
- *mass moment of inertia*
- *solving rigid body dynamics problems*
- *dynamics example: pulley with mass*

Rigid Body Kinematics

$$v_A = v_B + (\omega \times r_{A/B})$$

$$a_A = a_B + (\alpha \times r_{A/B}) + (\omega \times (\omega \times r_{A/B}))$$

- Useful Shortcuts for 2D planar motion

$$\alpha = \alpha\mathbf{k}$$
$$\omega = \omega\mathbf{k}$$
$$\mathbf{r} = r_x\mathbf{i} + r_y\mathbf{j}$$

$$\omega \times r = -r_y\omega\mathbf{i} + r_x\omega\mathbf{j}$$

$$\alpha \times r = -r_y\alpha\mathbf{i} + r_x\alpha\mathbf{j}$$

$$\omega \times (\omega \times r) = -r_x\omega^2\mathbf{i} - r_y\omega^2\mathbf{j}$$

Rigid Body Dynamics

Linear Motion:

$$\mathbf{F} = m\mathbf{a} = \frac{d(m\mathbf{v})}{dt}$$

sum of the forces is the time rate of change of linear momentum

Works for particles - and also works for rigid bodies if the acceleration is at the center of mass!

$$\mathbf{F} = m\mathbf{a}_{\mathbf{G}}$$

Rigid Body Dynamics

Rotational Motion:

$$\mathbf{M_G} = \frac{d(\mathbf{H_G})}{dt} = \dot{H}_G$$

sum of the moments is the time rate of change of angular momentum

about the center of mass

Rigid Body Dynamics

Rotational Motion:

$$\mathbf{M_G} = \frac{d(\mathbf{H_G})}{dt} = I_G \alpha$$

sum of the moments is the time rate of change of angular momentum

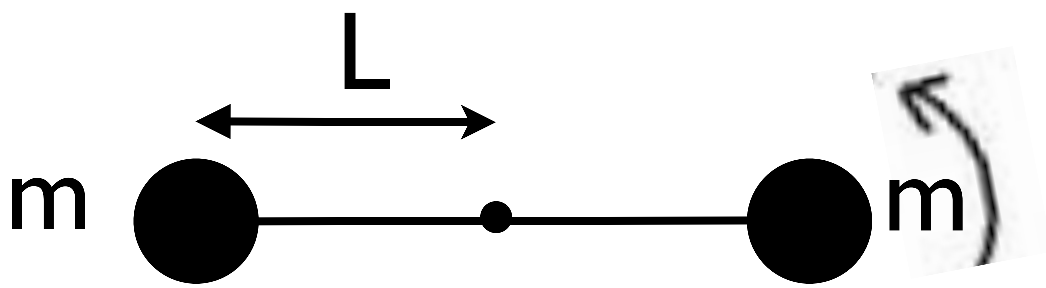
Mass Moment of Inertia:

$$I_G = \sum_i (m_i r_i^2) = \int_V (\rho r^2) dV$$

"Rotational Inertia"

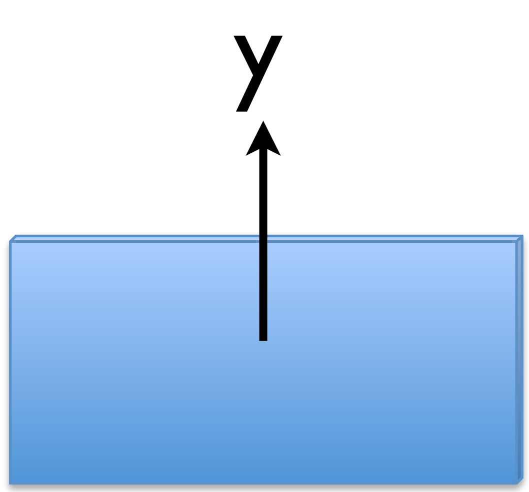
Resistance to angular acceleration

Mass Moment of Inertia

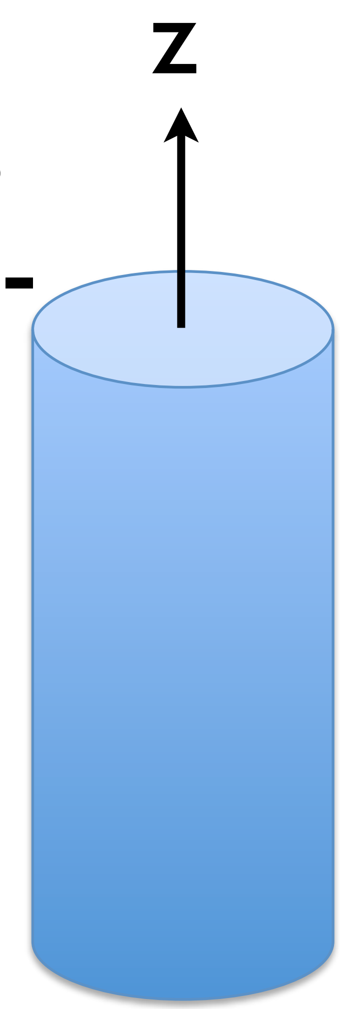


Two spherical masses connected by a massless bar

thin plate, height b, length c, rotating around y-axis

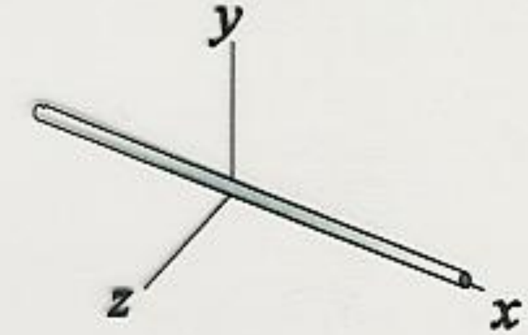


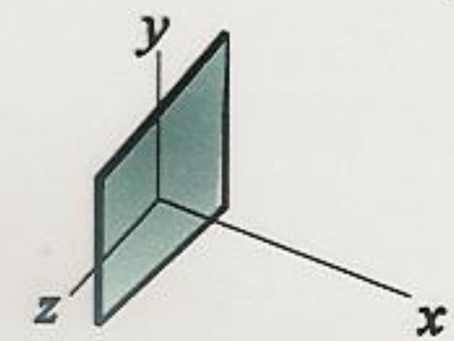
solid cylinder length L radius R, rotating around z-axis

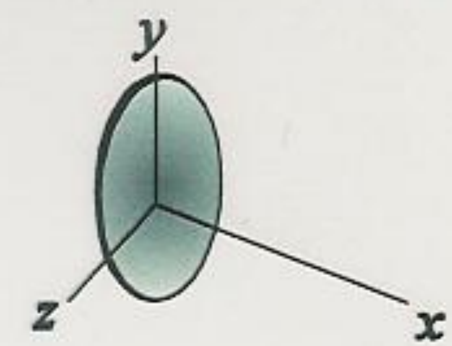


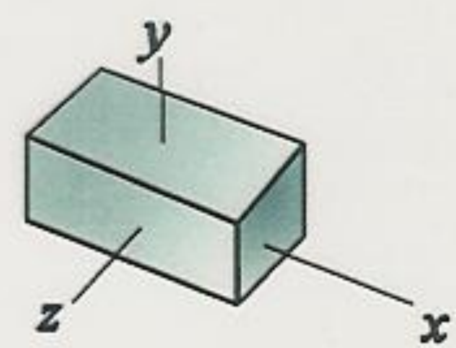
Principal Mass Moments of Inertia of Solid Geometrical Shapes

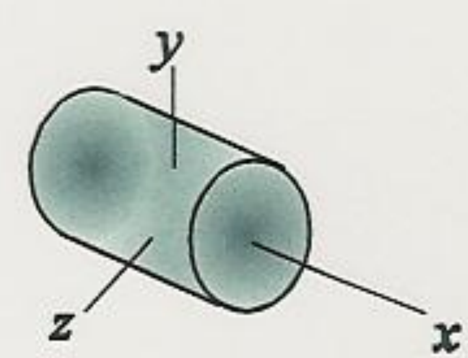
	I_x	I_y	I_z
Slender Rod m = mass, l = length of rod	0	$1/12\ ml^2$	$1/12\ ml^2$
Rectangular Plate m = mass, b = height of plate, c = width of plate	$1/12\ m(b^2+c^2)$	$1/12\ mc^2$	$1/12\ mb^2$
Thin Disk m = mass, r = radius of disk	$\frac{1}{2}\ mr^2$	$\frac{1}{4}\ mr^2$	$\frac{1}{4}\ mr^2$
Rectangular Prism m = mass, a = depth (x), b = height (y), c = width (z)	$1/12\ m(b^2+c^2)$	$1/12\ m(a^2+c^2)$	$1/12\ m(a^2+b^2)$
Circular Cylinder m = mass, l = length of cylinder, r = radius	$\frac{1}{2}\ mr^2$	$1/12\ m(3r^2+l^2)$	$1/12\ m(3r^2+l^2)$





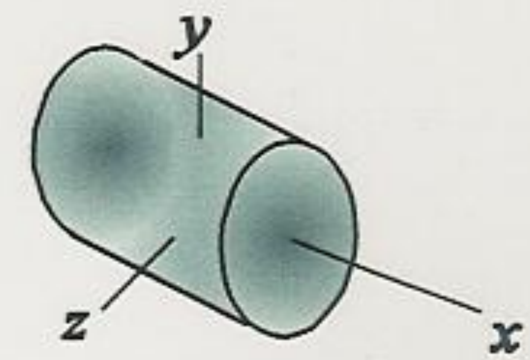






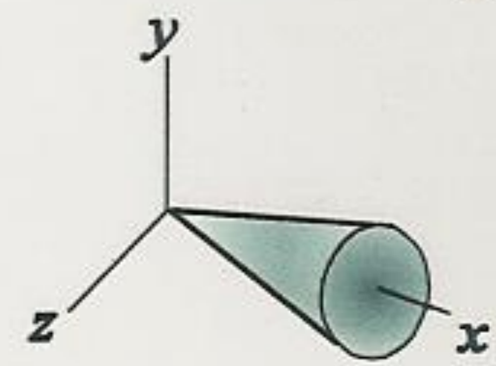
Elliptical Cylinder $1/12\ m(3c^2 + l^2)$ $1/12\ m(3b^2 + l^2)$ $¼\ m(b^2 + c^2)$

m = mass, l = length of cylinder (x), b = height/2 (y), c = width/2 (z)



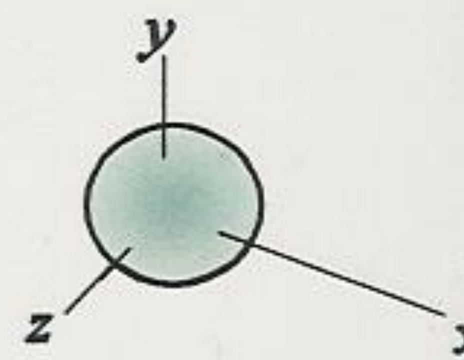
Circular Cone $3/10\ mr^2$ $3/5\ m(¼\ r^2 + l^2)$ $3/5\ m(¼\ r^2 + l^2)$

m = mass, l = length of cone, r = radius at base



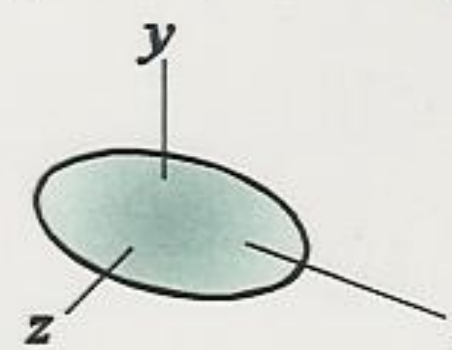
Sphere $2/5\ mr^2$ $2/5\ mr^2$ $2/5\ mr^2$

m = mass, r = radius

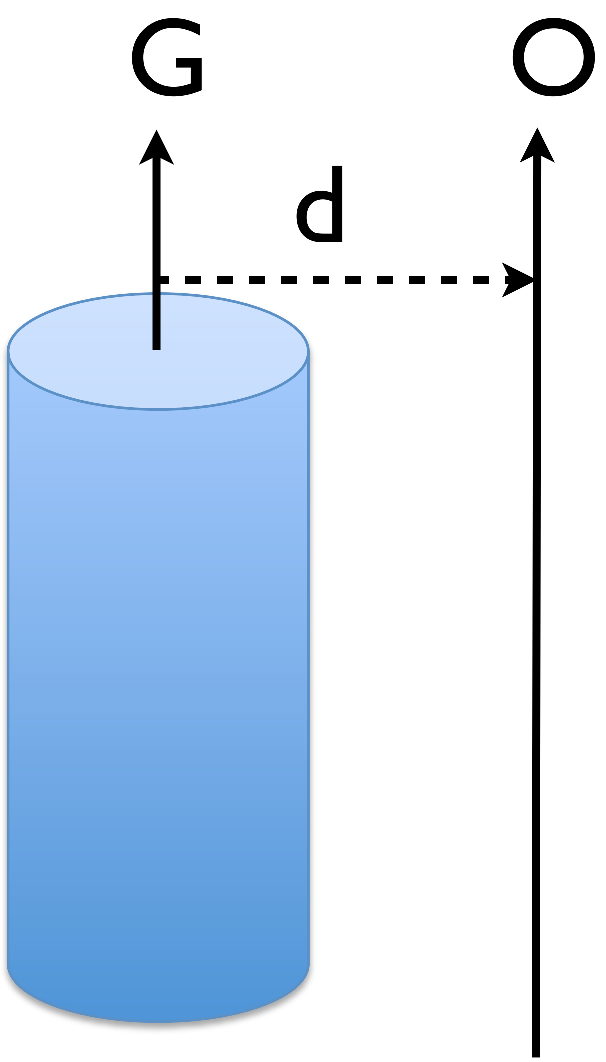


Ellipsoid $1/5\ m(b^2+c^2)$ $1/5\ m(a^2+c^2)$ $1/5\ m(a^2+b^2)$

m = mass, a = depth (x), b = height (y), c = width (z)



Parallel Axis Theorem



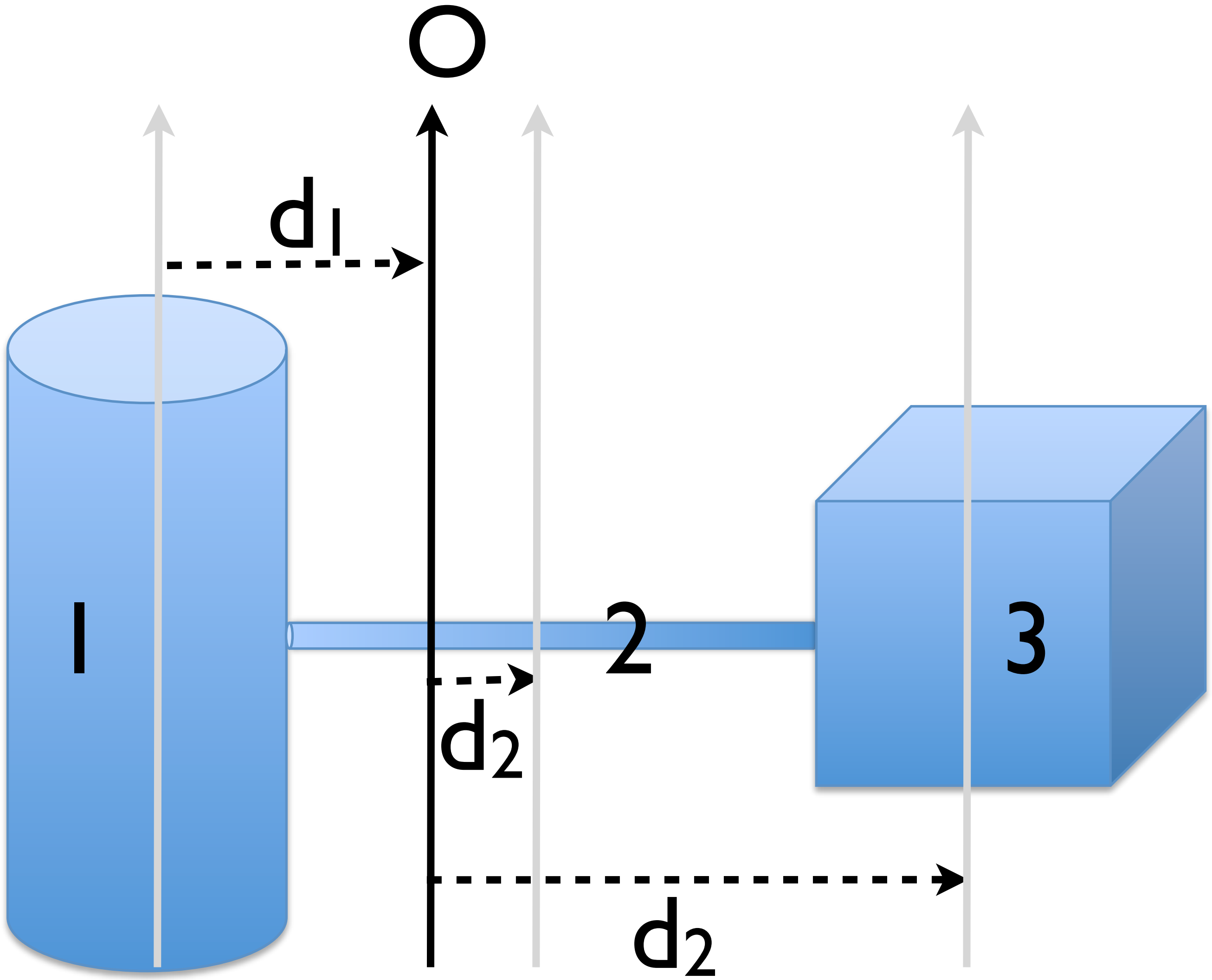
object rotating around axis "O" NOT through the CG

Find parallel axis through CG

$$I_O = I_G + md^2$$

I_G can read from tables

Composite Bodies



$$I_O = I_{G1} + m_1 d_1^2 + I_{G2} + m_2 d_2^2 + I_{G3} + m_3 d_3^2$$

I_G can read from tables

rigid body dynamics problems: 2D planar motion

- *Free Body Diagram!*
- *3 equations of motion:*

$$F_x = ma_x$$

$$F_y = ma_y$$

$$M_z = I\alpha$$

- *problem constraints*
- *mass moment of inertia calculation*
- *can we solve? if not, need more eqns:*
- *kinematics equations: connection between*

$$\alpha, \ \omega \ \text{AND} \ v, \ a$$

3. Analysis of Car Crash

A 2500 kg car crashes into a fixed barrier wall at speed v_c=5 m/s and is observed to rebound with a velocity of v_f=2 m/s. The total impact time is 0.5s. The passenger's torso and head is treated as a single rigid body, rotating about the point O as shown in the diagram below, where L is the distance from O to the center of mass. $I_G=1.2mL^2$ for the passenger.

refer to 2012 HW#7 for more details and solution