An Additive Exponential Noise Channel with a Transmission Deadline

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The Heroic Picture

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What can a cell tell the world?

The Heroic Picture

What can a cell tell the world?

What can a group of cells tell each other?

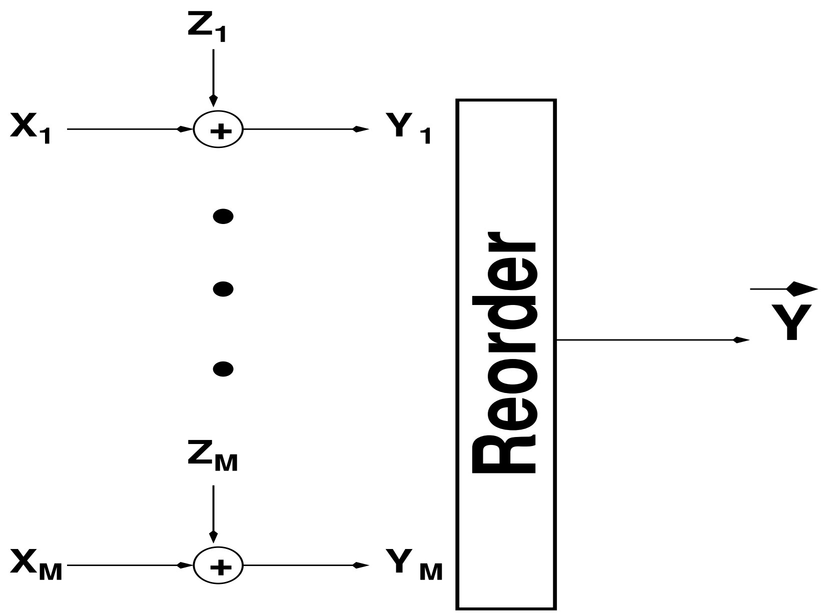
The Heroic Picture

What can a cell tell the world?

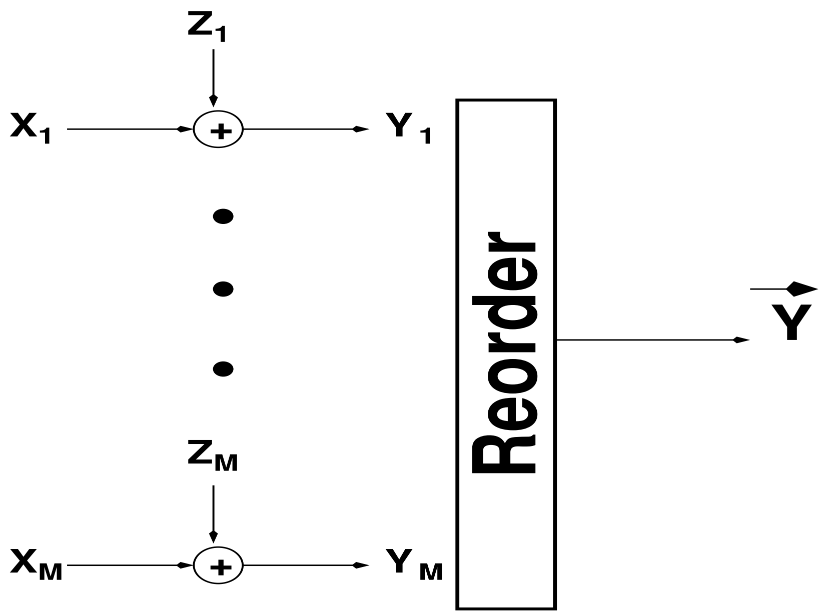
What can a group of cells tell each other?

Use IT *bounds* to avoid modeling morass

What Can a Cell Tell the World: abstraction

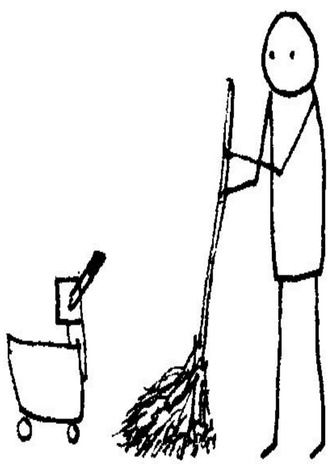


What Can a Cell Tell the World: abstraction



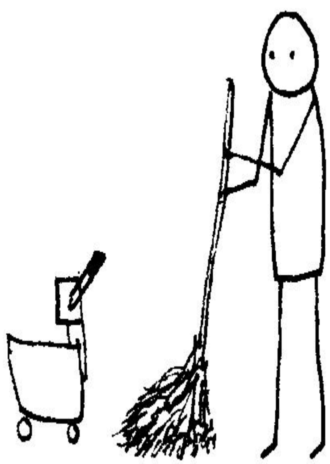
Simplistic but fundamental model

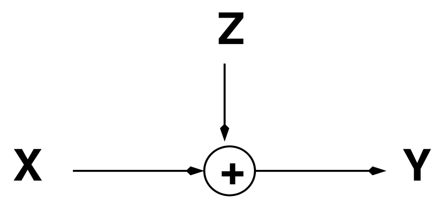
This Talk



This Talk

Subsubsubproblem





$$\max_{f_X()} I(X;Y)$$

Z exponential

$X \in [0, T]$

Structure of Output Density

$$f_Y(y) = \begin{cases} \lambda e^{-\lambda y} \int_0^y f_X(x) e^{\lambda x} dx & 0 \leq y \leq T \\ \lambda e^{-\lambda y} \int_0^T f_X(x) e^{\lambda x} dx & y > T \end{cases}$$

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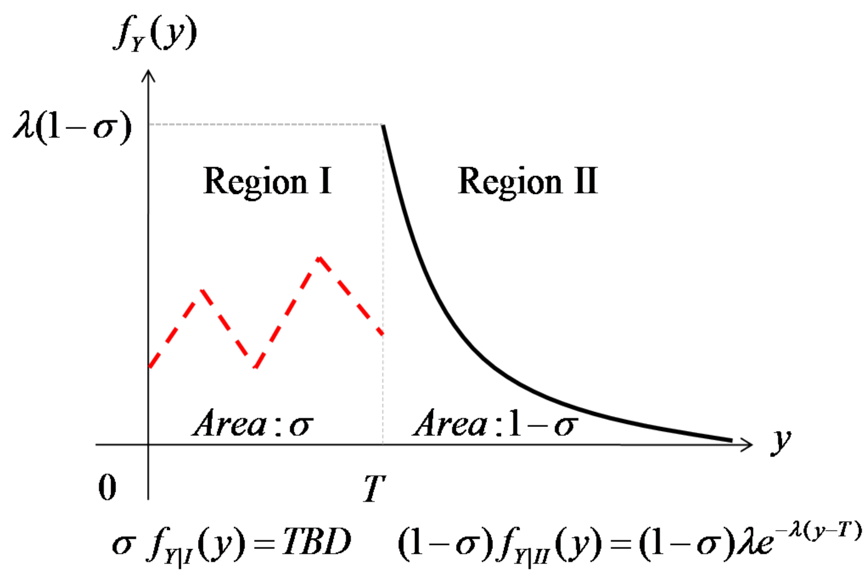

Entropy of Y

Alternatively:

$$f_Y(y) = \begin{cases} \sigma f_{Y|I}(y) & 0 \leq y \leq T \\ (1-\sigma) f_{Y|II}(y) & y > T \end{cases}$$

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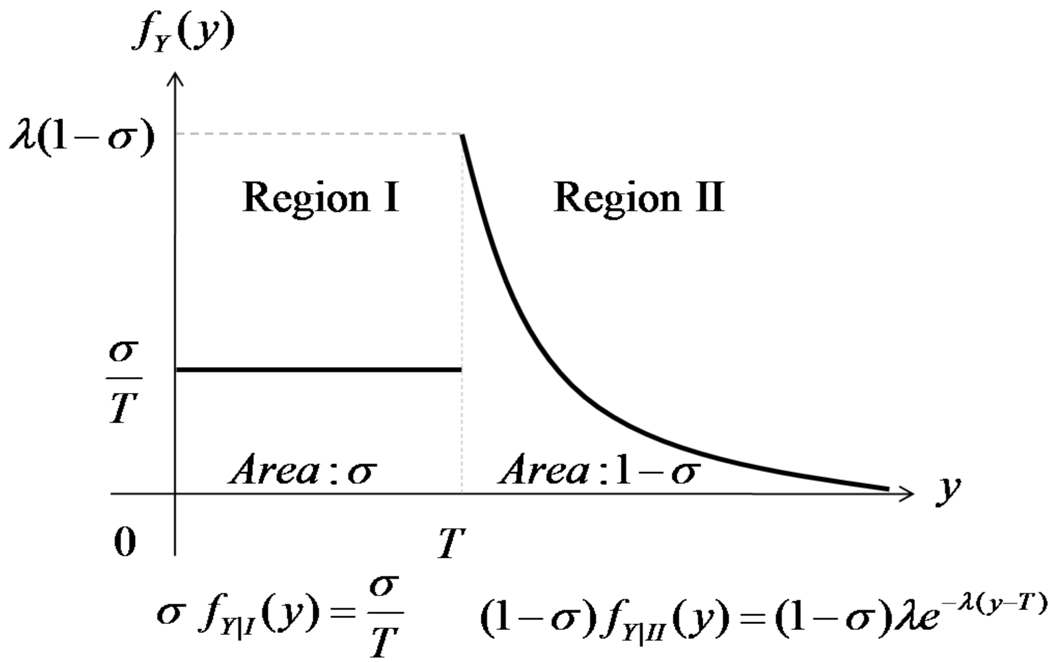
So,

$$h(Y) = \sigma h(Y|I) + (1-\sigma) h(Y|II) + H_B(\sigma)$$

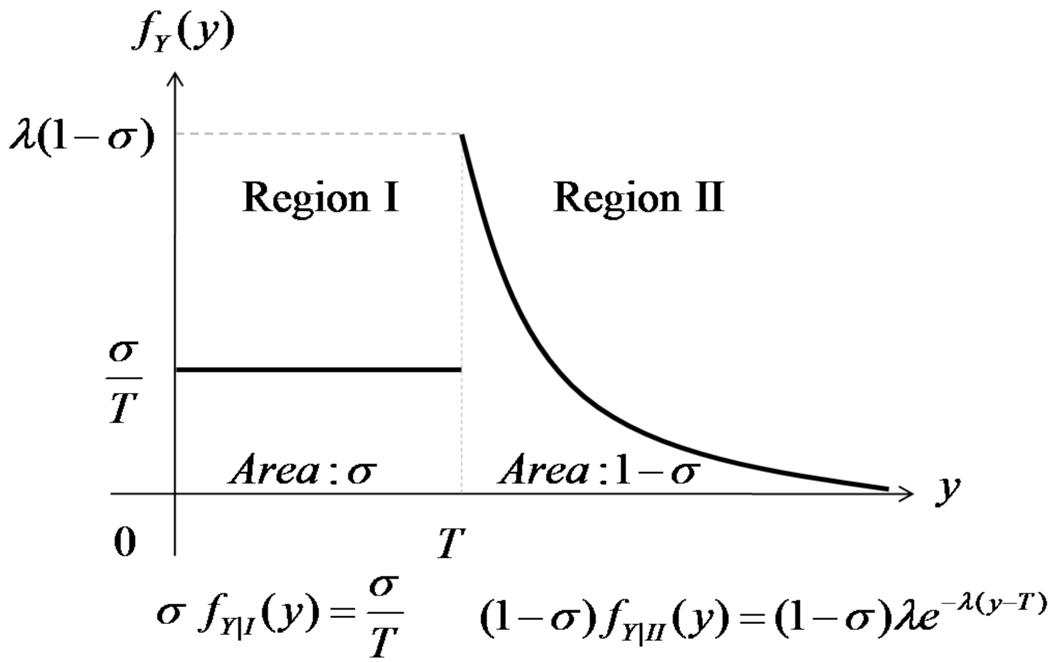
$$h(Y|II) = 1 - \log \lambda$$

Maximizing $h(Y)$ Step 1: fix σ



$$\max h(Y|I) = \log T$$

Maximizing $h(Y)$ Step 1: fix σ

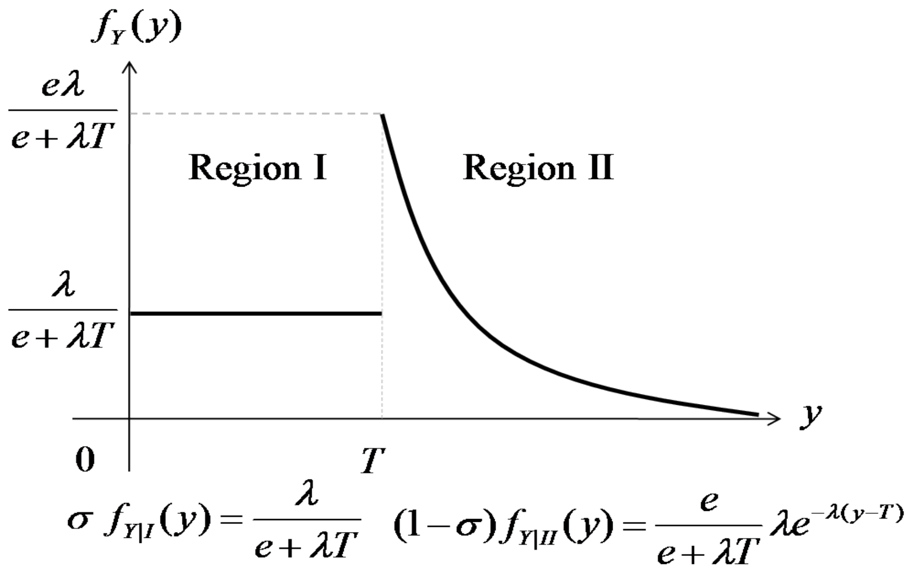


$$\max h(Y|I) = \log T$$

So

$$h(Y) = \sigma \log T + (1-\sigma)(1-\log\lambda) + H_B(\sigma)$$

Maximizing $h(Y)$ Step 2: optimize in σ



$$\arg\max_{\sigma} h(Y) = \frac{\lambda T}{e + \lambda T}$$

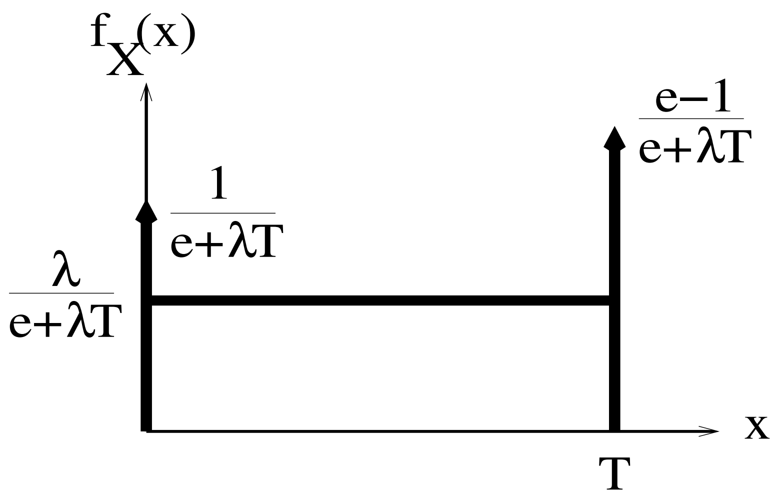
$$\max_{f_X} h(Y) \leq \log\left(\frac{e + \lambda T}{\lambda}\right)$$

Nailing It Down

$$h(Y) = \log\left(\frac{e+\lambda T}{\lambda}\right)$$

when

$$f_X(x) = \frac{\lambda}{e+\lambda T}(u(x) - u(x-T)) + \delta(x)\frac{1}{e+\lambda T} + \delta(x-T)\frac{e-1}{e+\lambda T}$$



$I(X;Y)$ Factoids

For exponential first passage density f_Z with mean $\frac{1}{\lambda}$:

$$\max_{f_X} I(X;Y) = \log\left(1 + \frac{\lambda T}{e}\right) \tag{1}$$

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Suppose $E[Z] = \frac{1}{\lambda}$ but Z is not exponential:

$$\max_{f_X} I(X;Z+X) \geq \log\left(1 + \frac{\lambda T}{e}\right)$$

(easy proof in paper – stolen from Venkat & Sergio)

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Exponential hurts most

(analogous to Gaussian)

What It All Means?

YAACCD:

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YAACCD: yet another additive channel capacity derivation

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BUT: allows bounds to be derived