How Well Can We Know A MIMO Channel?

1 Introduction

Wireless channels are notorious for their spatiotemporal variation – so much so, that the usual recourse is to assume stochastic models [1–3] and devise signaling methods which work well under uncertainty [4–6]. This state of affairs is especially frustrating for multiple input multiple output (MIMO) channels because of the large potential gains possible were only the channel well known [4, 7–11]. Furthermore, MIMO channels can be exploited for better inter-user isolation in multi-user systems [7, 12, 13] and even, potentially, perfect wireless secrecy [6] – a seeming oxymoron.. Precise channel knowledge is essential for such applications. It is these tantalizing benefits which prompt us to ask the basic question: how well can we know a MIMO channel?

The underlying physics of wireless channel composition is what makes these sorts of questions so interesting – stochastic variation of wireless channels is caused by the stochastic movement of scatterers and transceivers. Scatterers could be vehicles, buildings, lampposts, trees, animals, people, sometimes even the atmosphere – anything that interacts with radio waves and moves. And it is exactly this dependence of the channel on the motion of macroscopic objects which suggests that knowing the wireless channel and tracking its changes might not be the seeming fool's errand which has historically caused wireless theorists to collectively throw up our hands and invoke probability theory.

Specifically, the motion of the macroscopic objects which cause scattering is constrained by the energy available to move them. In particular, we can show that *unpredictable* motion is limited owing to the energy necessary to change the momentum of an object. Thus, a skyscraper may sway, a vehicle may speed and a person may shift position, but all will do so in a relatively predictable manner – which implies that the *information rate* necessary to specify their positions could be relatively small.

The information and information rates which are necessary to specify relevant constituent parts of the channel constitute exactly an upper bound on the amount of information that must be extracted from measurements to specify the channel. Thus, by first deriving bounds on the information necessary to specify and track relevant channel constituents and then comparing them to the amount of information that can be extracted from channel interrogation, we can determine *precisely* whether knowing a fading MIMO channel well is possible or impossible. We suspect that because constituents of most scattering environments are relatively massive objects and energy to move them stochastically is limited, the information rates necessary to track changes and disseminate them appropriately could be manageable. Establishing exactly when the channel can be known with an eye toward the benefits which can accrue from such knowledge is our aim.

To do so, we must first quantify the effects of inaccurate channel knowledge and establish just how well the channel must be known in order to realize the full benefits of MIMO systems. We will explore these issues somewhat anecdotally in this proposal to obtain a feel for the necessary precision, but our ultimate goal is to determine how we may specify the channel state in bits – essentially a quantization problem.

We must then consider the modeling of scattering channels at a physical level as an assortment of spatially distributed scattering objects (perhaps grouped into "scattering centers" [14]) whose ensemble properties and evolution we wish to track. We will argue that energy considerations constrain the stochastic motion of scattering center constituents as well as transceivers to allow more directed estimation of channel parameters from available probe measurements. Or more simply put, we will argue that our approach, driven by energy constraints on objects in motion, can bound the channel tracking search space and thus help with more efficient channel state evolution prediction.

Overall we seek to:

- Quantify the necessary channel state information (CSI) for accuracy levels which enable benefits such as increased rate, mutual interference mitigation and perfect secrecy in MIMO systems.
- Understand channel variation as a function of energy bounds on the physical mobility of constituent scattering objects and transceivers, thereby providing an upper bound on the entropy rate of CSI.
- Determine when and how such models might be acquired blindly (and/or under pre-existing generic classifications of scatterer types) from channel probes including potential augmentation with dedicated sensor measurements.
- When channel state prediction is possible, demonstrate its feasibility through analysis and simulation.

2 Communication Channel Model

We will use the generic signal space vector-channel model

$$\mathbf{r} = \mathbf{G}\mathbf{u} + \mathbf{w} \tag{1}$$

where received vector $\mathbf{r}$ and noise vector $\mathbf{w}$ are N-dimensional, and transmit vector $\mathbf{u}$ is M-dimensional. The gain matrix $\mathbf{G}$ is thus of dimension $N \times M$. The assumption with any such model is that all transmitters and receiver waveforms lie in some common signal space defined by some common set of orthonormal basis functions. Usually, owing to an assumption of channel linearity, time-invariance and synchronization, these basis functions are sinusoids. However, the signal space description could also be time-based with different dimensions corresponding to different time samples or whatever convenient basis set is available.

Here we will always assume $\mathbf{w}$ is Gaussian, though not necessarily white, and that the gain matrix $\mathbf{G}$ is random in a way reflected by the physics of the particular channel and constant over the signaling interval. That is, $\mathbf{G}$ is a specific "channel instance" during each signaling interval for which equation (1) applies. The sequence of channel instances is assumed to be some ergodic though not necessarily stationary stochastic process. For a known channel $\mathbf{G}$ and Gaussian noise $\mathbf{w}$ with covariance $\mathbf{W}$, the channel capacity is given by [4, 8–11]

$$C(\mathbf{G}) = \max_{\mathbf{R_u}, \text{Trace}[\mathbf{R_u}] \le P} \frac{1}{2} \left(\log \left| \mathbf{G}\mathbf{R_u}\mathbf{G}^H + \mathbf{W} \right| - \log |\mathbf{W}| \right) \tag{2}$$

where $\mathbf{R_u} = E[\mathbf{u}\mathbf{u}^H]$, P is the power available for signaling and the u_i which comprise $\mathbf{u}$ are zero mean and jointly Gaussian. The optimal $\mathbf{R_u}$ is the usual water-filling solution in the right-eigenspace of $\mathbf{W}^{-1/2}\mathbf{G}$, waterfilled over the inverse of it magnitude-squared non-zero singular values. The total capacity is then $C(\mathbf{G})$ [4] averaged over all channel instances

$$C = E_{\mathbf{G}} \left[C(\mathbf{G}) \right] \quad . \tag{3}$$

One of our overall goals will be to establish a mapping from scattering center characteristics to explicit channel gains, or where a deterministic mapping is too difficult, more refined probability distributions on channel gains than those which are generally assumed without detailed knowledge of the underlying channel physics. The details of our channel modeling approach are considered in section 4 along with an illustrative example. However, we must first consider the issue of performance in the face of channel uncertainty.

3 CSI and MIMO

Most current work on channel state information for MIMO systems asks questions about the rates achievable, outage probabilities and other performance metrics when channel state is known to the transmitter, to the receiver, both – or if neither, what performance is possible when the channel state distribution is known (see [4] for a recent survey). Such studies provide powerful outer bounds, but do not quite address on of the questions we hope to pose – how does one usefully quantify channel state. This problem is a bit difficult in that the usual measures of accuracy – such as variance bounds on the entries g_{ij} of a gain matrix $\mathbf{G}$ – may not be as useful as in single input single output (SISO) cases.

For instance, knowing the precise value of a nearly zero gain in a MIMO system is simply not that important when channels with much stronger gains are available. More critical in this case would be telling the difference between good and bad signaling dimensions quickly. Furthermore, although the specific gains are important in a MIMO system, so is the structure of the vector space they express. Finding ways to usefully measure and quantify CSI is therefore part of this proposed work, although the usual approach is to find minimum mean square error estimates of channel matrices [15–21].

That notwithstanding, we will start our explorations with the usual model typically used for MIMO capacity with estimation error problems [20, 21]

$$\hat{\mathbf{G}} = \sqrt{1-\sigma_e^2}\mathbf{G} + \mathbf{Q} \tag{4}$$

where $\mathbf{Q}$ is a zero mean matrix with independent identically distributed random entries of some "error variance" σ_e^2. $\mathbf{G}$ is renormalized so that $\hat{\mathbf{G}}$ also has unit variance entries. This type of assumption is reasonable for many channel interrogation methods.

3.1 A Direct Approach to Channel Gain Matrix Quantization

One might first approach the channel state quantization problem by simply using equation (2) in conjunction with equation (4) to compute the average variation between $C(\mathbf{G})$ and $C(\hat{\mathbf{G}})$ for various values of σ_e^2. Of course, this approach completely ignores the assumption that the channel state information is limited in some fundamental way, but is reasonable from the analytic standpoint of determining neighborhoods around gain matrices. To this end, we provide a plot of the variance of

$$\Delta G = \frac{C(\mathbf{G}) - C(\hat{\mathbf{G}})}{C(\mathbf{G})} \tag{5}$$

derived from Monte Carlo simulations wherein $\mathbf{G}$ was chosen to have unit variance, zero mean circularly Gaussian entries and $\hat{\mathbf{G}}$ was derived from $\mathbf{G}$ as in equation (4). The results are provided in FIGURE 1 for white noise ($\mathbf{W} = \mathbf{I}$) and different power levels $P = 1$, 10 and 100. Using the typical two standard deviation measure of certainty, a value of $\sigma_e^2 \approx 0.05$ for $P = 1$ and $\sigma_e^2 \approx 0.1$ for $P = 10$ and $P = 100$ might seem reasonable to define a 10% capacity neighborhood about a given gain matrix. In contrast, a 1% ΔG neighborhood would require a much more stringent $\sigma_e^2 \approx 0.001$ for $P = 1$, 10 and 100.

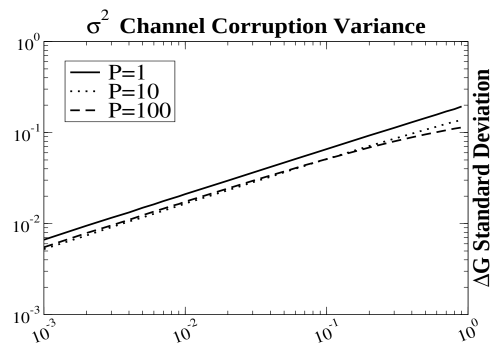

Figure 1: Standard Deviation of ΔG vs. σ_e^2 for a 4×4 System. Power Budgets $P = 1$, 10 and 100 shown.

3.2 An Information Theoretic Approach to Gain Matrix Quantization

The previous section compared the capacities obtainable with different gain matrices whose elements differed from one another by some mean square amount. The capacity calculations assumed the gain matrices were known exactly at the transmitter and receiver – which is unsatisfying since the issue is that channel state information is fundamentally limited. Thus, we need another measure of performance penalty – one derived specifically for imperfect channel knowledge. Recently it has been shown [21] that the lower bound on capacity $\tilde{C}$ of an estimated channel is given by

$$\tilde{C} = E\left[\max_{\hat{\mathbf{R}}_{\mathbf{u}},\mathrm{Trace}[\hat{\mathbf{R}}_{\mathbf{u}}]\leq P} \frac{1}{2}\left(\log\left|\frac{1}{1+\sigma_e^2 P}\hat{\mathbf{G}}\hat{\mathbf{R}}_{\mathbf{u}}\hat{\mathbf{G}}^H + \mathbf{I}\right|\right)\right] \tag{6}$$

where the noise covariance $\mathbf{W}$ is assumed white ($\mathbf{W} = \mathbf{I}$). The upper bound is slightly more complex, but was shown to be reasonably tight for Gaussian signaling [21], so we will use equation (6) as our measure of performance under channel uncertainty.

With a comparison method using equation (2) vs. equation (6) and an implicit gain matrix "closeness measure" based on equation (4) we can once again roughly quantize channel states. First we quantize capacity into discrete ranges. Then we evaluate how large σ_e^2 must be to cause $\tilde{C}$ to differ by at least one quantization level from C.

For example, under the channel model of equation (1), consider a channel gain matrix $\mathbf{G}$ with zero mean, unit variance and circularly symmetric complex Gaussian entries. Assume $\mathbf{w}$ is a zero mean white Gaussian noise vector, each entry with unit variance. The capacity of this channel following equation (2) is

$$C(\mathbf{G}) = \max_{\mathbf{R}_{\mathbf{u}},\mathrm{Trace}[\mathbf{R}_{\mathbf{u}}]\leq P} \frac{1}{2}\log\left|\mathbf{G}\mathbf{R}_{\mathbf{u}}\mathbf{G}^H + \mathbf{I}\right| \quad . \tag{7}$$

For each random $\mathbf{G}$ we assume corruption as in equation (4) where $\mathbf{Q}$ has zero mean circularly symmetric independent Gaussian entries like $\mathbf{G}$, but with variance σ_e^2. We then calculate $\tilde{C}$ as in equation (6) using Monte Carlo methods. The result is FIGURE 2 where we plot estimation error fraction

$$\Delta C = \frac{C - \tilde{C}}{C} \tag{8}$$

for a 4×4 system as a function of error variance σ_e^2 using power budgets $P = 1$, 10 and 100. Setting our capacity "quantum" to 10%, we see that the channel estimation accuracy necessary to maintain $\tilde{C}$ within the margin for $P = 100$ is about $\sigma_e^2 = 0.005$ while for $P = 1$, the necessary accuracy drops to $\sigma_e^2 = 0.2$. Were a 1% quantum desired, then the error tolerance levels drop significantly.

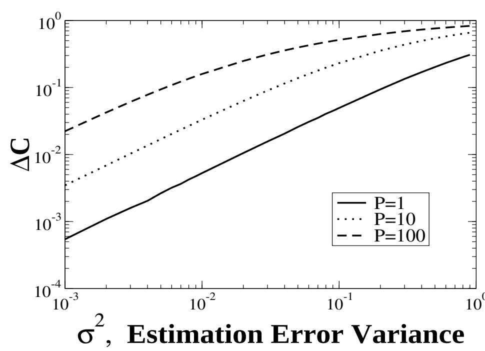

Figure 2: Fractional Capacity Difference ΔC vs. σ_e^2 for a 4×4 System. Power levels as shown.

3.3 MIMO Channels and Secrecy

The very thing which makes wireless systems so attractive – tetherless access to information – is what makes them seem so unavoidably insecure. Once radiated, wireless transmissions might be detectable by anyone with a suitable receiver. This perceived vulnerability is obvious and the most common practical response is to provide some level of encryption over the wireless link [22, 23]. Other responses include attempts to make the path between transmitter and receiver difficult to identify, as in meteor shower channels [24] or keeping an eavesdropper from receiving any useful signal energy as with narrow beam communication using directive antennas.

Encryption is the only option when transmission of information must be done in the open and is accessible to everyone, however, its unavoidable weakness lies in the necessity of keeping the key(s) secret. In contrast, information theory tells us that *perfect secrecy* is attainable if we can (essentially) ensure that the channel between desired users is "less noisy" than that to an eavesdropper [6, 25, 26]. Thus, the common-sense approach to wireless security would seem to be encryption since guaranteeing that the eavesdropper's channel is inferior to that of the intended receiver in scattering terrestrial wireless channels seems an almost impossible task.

Or is it?

This question has recently been asked for MIMO channels [6]. In scattering environments, there exist many paths between a given transmitter and receiver, and these paths could be very different depending on the geographic placement of transceivers. Thus, given communicators Alice and Bob and a potential eavesdropper Eve, there could be signal paths (eigenmodes of the channel) between A and B which are are poorly received by E. Good reception by B and poor reception by E can be shown to provide a nonzero *perfectly secret* capacity between A and B [25, 27], and efficient codes exist to exploit this capacity [26].

Previous work [6] approaches this topic by assuming various degrees of channel knowledge available to A and B while poor channel knowledge is available to E. For complete channel knowledge between A and B and poor channel knowledge between A and E, the secret capacity between A and B is clearly maximum. However, the fundamental premise of [6] – E's lack of channel knowledge – is in some sense antithetical to the usual ethos of security. That is, information not specifically blocked by various (cryptographic or physical) barriers is assumed universally known.

We note that if Eve has access to all the radiated energy in the environment, nothing can be kept perfectly secret and cryptography – as opposed to information theoretic perfect secrecy – seems the only recourse. However, if Eve's resources for interrogation of the environment are limited, a number of questions are raised: Assuming all channels are known, but Eve does not have access to all radiated energy, what is the likelihood that there exist signal dimensions in Alice and Bob's channel which can support secret communication which excludes Eve? What can Eve know about the channel between Alice and Bob? How does channel uncertainty affect secret communication rates between Alice and Bob?

These sorts of fundamental questions, which are at heart channel characteristic and channel measurement questions, are squarely within the purview of our proposed research. However, without underlying physical channel models, to which we turn next, at least two of these questions – those dealing with channel sensing – are moot.

4 Can the Channel State Be Known?

In previous sections we have argued that knowing the channel can provide various performance boosts for MIMO systems and potentially enable applications like perfect wireless secrecy [6] which are difficult if not impossible without good channel information. These potential benefits prompt us to ask just how difficult channel identification and tracking is at a fundamental information theoretic level. To answer such questions requires some sort of channel model.

In a typical depiction, a MIMO channel is shown as a cloud in which something called "scattering" occurs. This scattering is usually summarized by a stochastic sequence of channel matrices $\mathbf{G}_i$. A large amount of work has been devoted to characterizing this stochastic process in a variety of situations with the most prevalent being complex circularly symmetric zero mean Gaussian channel gains with or without correlation between entries [4]. Owing to the previously discussed strong improvements possible when channel state is known, a large amount of work has also been devoted to estimating MIMO channel matrices ([15, 17–19, 28, 29] and references therein). In work roughly similar to the approach we will take, a "parametric" method of modeling and estimation was used with specified number of rays along with separate delay and phase estimates since delay profiles vary much more slowly than phase for exactly the reasons we posit here – macroscopic objects move slowly [30].

However, we are aware of no work which compares the channel state information rate available from interrogation to the underlying information and information rates which are characterized by the channel physics. Thus, our goal is to more carefully model the inner workings of the channel in the hope that somewhat more detailed knowledge will lead to practical methods of learning the channel and its evolution.

It seems reasonable early on to avoid overly complex models since these may not provide useful analytic insight. To this end, consider FIGURE 3 which though still a cartoon of a scattering channel, is slightly more detailed in that it depicts an assortment of scattering objects in motion. The objects could have different scattering characteristics, but for now we will assume identical characteristics for simplicity. The channel gain between a set of antennas

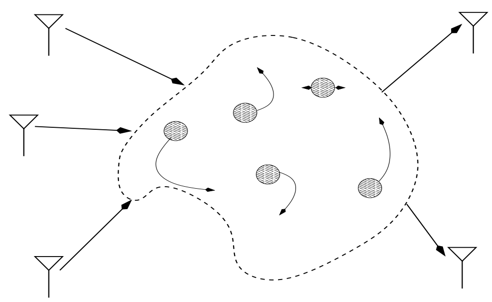

Figure 3: Detail of MIMO Scattering Channel With Mobile Components.

over the channel is expressible as a superposition of scatterer effects and depends solely on transceiver and scatterer positions. Scatterer orientation could also be used as a state variable, but for our purposes now, position will suffice.

Scatterer movement results in channel gain matrix variation and thereby performance variation. Following the ideas introduced in section 3, we can determine a useful quantization of channel gain matrices which through the channel model implies a useful quantization of scatterer position. In this way we can begin to specify the amount of information necessary to "know a channel instance." In addition, we can also obtain some feel for channel coherence times – the time a given channel instance is in force – as a function of mean scatterer velocities, and this can be vetted against measurements of real channels as a check on the validity of the modeling.

With channels and scatterer positions quantized, we can then ask how much information is conveyed about the channel by interrogation over a coherence time. Thus armed, it is simple in principle to compare the channel interrogation information rate to the ratio of channel state information to coherence time. An insufficiently large channel interrogation information rate implies the channel state instance is unknowable on average, and some means of increasing the rate at which channel information can be learned is necessary – perhaps through dedicated channel sensing infrastructure enhancements. Conversely, if the interrogation rate is sufficiently large, then the channel is theoretically knowable.

The next step is to consider stochastic motion of scatterers. Through the channel model this leads to a stochastic sequence of channel instances. However, since scatterer positions map deterministically to channel gains, the information processing theorem [31] restricts the entropy rate of the channel gain matrix process to be less than or equal to the entropy rate of the scatterer mobility process. This channel entropy rate can in principle be derived from the mobility entropy rate. As before, the channel entropy rate can be compared to the rate at which channel information can be extracted from interrogation. If the interrogation information rate is larger than the channel entropy rate, then the channel can be known. If not, then the channel is unknowable without providing an additional means of channel sensing.

These ideas which form the basis of our approach are worth expanding upon with more technical detail and with reference to the picture provided in FIGURE 4. First we define

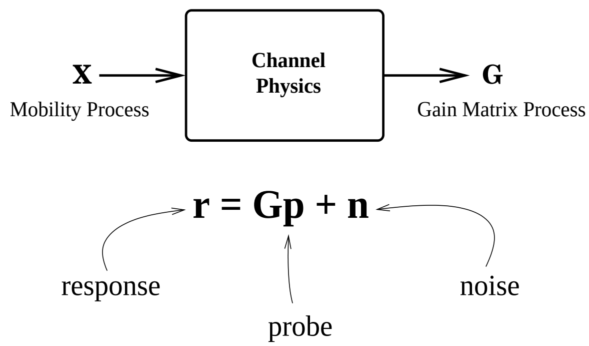

Figure 4: Representation of Scatterer Mobility Information Flow

a quantized scatterer mobility process $\mathbf{X}$ which has some entropy $H(\mathbf{X})$ and entropy rate $\mathcal{H}(\mathbf{X})$. This process serves as input to the channel physics which produces a corresponding gain matrix $\mathbf{G}$. By the information processing theorem, $H(\mathbf{X}) \geq H(\mathbf{G})$ and $\mathcal{H}(\mathbf{X}) \geq \mathcal{H}(\mathbf{G})$. So all we need to know about the channel can be specified using at most $H(\mathbf{X})$ bits and the channel variation can be tracked using at most $\mathcal{H}(\mathbf{X})$ bits per unit time.

As previously mentioned there is a large literature on channel estimation, but we will at first opt for the simplest case where we assume blind channel interrogation of the form

$$\mathbf{r} = \mathbf{G}\mathbf{p} + \mathbf{n} \tag{9}$$

where $\mathbf{r}$ is the received vector, $\mathbf{p}$ is the probe vector with some power $|\mathbf{p}|^2$, and $\mathbf{n}$ is the measurement noise which we will assume is zero mean circularly symmetric white noise with variance 1. We will also assume no feedback between successive "sounding" epochs. Since the number of parameters to be specified in $\mathbf{G}$ is generally larger than the dimension of the probe vector, we can assume multiple "soundings" and represent the measurement set as a matrix $\mathbf{R}$, whose columns are the individual soundings. The probes are assumed known at the receiver. The matrix $\mathbf{G}$ is assumed random with some distribution so that the amount of information $\mathbf{R}$ gives us about $\mathbf{G}$ is simply

$$I(\mathbf{G}; \mathbf{R}) = H(\mathbf{R}) - H(\mathbf{R}|\mathbf{G}) \tag{10}$$

which we define as the interrogation rate for channel $\mathbf{G}$

$$\mathcal{R}(\mathbf{G}) \equiv I(\mathbf{G}; \mathbf{R}) \tag{11}$$

$\mathcal{R}(\mathbf{G})$ is an upper bound on the amount of information we can learn about the channel $\mathbf{G}$ per interrogation. Our approach will be to make comparisons between the interrogation rate and the underlying entropies $H(\mathbf{G})$ and $H(\mathbf{X})$ to determine whether the channel can possibly be known or not. This approach is roughly summarized by equation (12)

$$\mathcal{R}(\mathbf{G}) \underset{\text{cannot}}{\overset{\text{can learn}}{\gtrless}} \frac{1}{N_I} H(\mathbf{G}) \tag{12}$$

where N_I is the number of interrogations (static channel) and equation (13)

$$\mathcal{R}(\mathbf{G}) \underset{\text{cannot}}{\overset{\text{can track}}{\gtrless}} \mathcal{H}(\mathbf{G}) \tag{13}$$

4.1 Calculating Channel Gains

We will take the simplest possible approach to calculating channel gains from the model of FIGURE 3 to avoid obscuring our main points. Each sphere will be treated as a point which scatters energy from the transmitting antennas. We will not consider secondary scattering (scatter on one sphere induced by the scatter impinging from another sphere). We will also assume there exists no direct path between the transmitting and receiving antennas.

The position of the k^{th} sphere is $\mathbf{x}_k$, and the position of the n^{th} antenna is $\mathbf{y}_n$. The propagation paths between transmit antenna m and receive antenna n are composed of two "legs" – one from antenna m to a scatterer k and then from scatterer k to antenna n. We will not worry about angle of arrival – which determines the effective scattering (radar) cross section from the perspective of the receiving antenna – but will assume isotropic scatter from each sphere whose intensity is solely dependent on the path length between the (isotropic) transmitting and antenna and the scattering sphere. We will also assume that phase shifts depend strictly on path length as opposed to the detailed characteristics of the scatterers. Again, this is not a terrible assumption given our identical scatterer model. Finally we assume scatterers and antennas are in each other's far field.

Under these assumptions, we can readily calculate the gains g_{mn} between antennas. First we calculate the first leg length $d_m(k) = |\mathbf{x}_k - \mathbf{y}_m|$ and then the second leg length $d_n(k) = |\mathbf{x}_k - \mathbf{y}_n|$. Antenna aperture and radar cross section of scatterers add constants to our simple model which we can ignore here as fixed owing to the uniformity of the scattering environment and isotropic scattering and antennas. Thus, we can calculate the the gain of the path between antennas m and n using scatterer k as

$$g_{mn}(k) = e^{j\phi_{nm}(k)} r_{mn}(k) \tag{14}$$

where

$$r_{mn}(k) = |g_{mn}(k)| = \frac{A}{d_m(k)} \frac{\alpha}{d_n(k)} \tag{15}$$

and

$$\phi_{mn}(k) = 2\pi(d_m(k) + d_n(k))/\lambda \tag{16}$$

where λ is the wavelength and A and α are proportionality constants related to antenna gains and object scattering cross section. Since the antennas are all assumed identical as are the scatterers, we will set A and α to one with no loss of generality. The total complex gain g_{mn}

between antenna m and n is then given by

$$g_{mn} = \sum_k r_{mn}(k) \cos \phi_{nm}(k) - j \sum_k r_{mn}(k) \sin \phi_{nm}(k) \tag{17}$$

Given scatterer positions and antenna positions we can now calculate the complex gains for our toy MIMO channel. We can then perturb the scatterer positions and see how the channel gains are affected and via the discussion in section 3, the capacity.

Measuring all distances in units of wavelength, we place our two transmit antennas at $(-\frac{D}{2}, \frac{1}{2})$ and $(-\frac{D}{2}, -\frac{1}{2})$ so that they are one wavelength apart. The two receiver antennas are placed at $(\frac{D}{2}, \frac{1}{2})$ and $(\frac{D}{2}, -\frac{1}{2})$ where $D \gg 1$. The scatterers are placed randomly in within a square with corners $(-L, -L)$ and (L, L). For $D = 1000$ and $L = 300$ and each random placement of 8 spheres, we calculate the 2×2 gain matrix $\mathbf{G}$ and summarize the results in FIGURE 5 – a scatterplot where g_{11} is plotted as a point in polar coordinates as shown. Evaluation of gain histograms over many trials reveals that gains g_{ij} approximately follow the

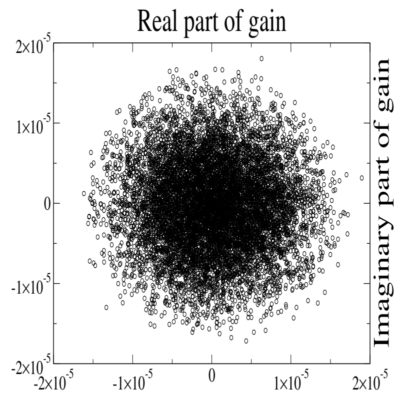

Figure 5: Channel Gain Scatterplot for An 8-Sphere Channel. 1000 trials.

circularly symmetric Gaussian distribution typical of Rayleigh fading – as should be expected in a highly scattering environment [1,32].

We now choose particular random arrangements of spheres and then repeatedly "jiggle" their coordinates using i.i.d Gaussians of variance σ_x^2. In this way we can build up a statistical picture of how mean scatterer location uncertainty maps to mean gain matrix uncertainty. Using Monte Carlo simulation, a typical plot of mean square difference (per element) between the original and position-perturbed gain matrices (normalized by the gain variance of the channel) as a function of mean square position uncertainty σ_x^2 is shown in FIGURE 6. In section 3 we found that a gain variances in the range of 0.001 to 0.1 would admit performance penalties in the range of 1% to 10%. The corresponding position uncertainties seen in FIGURE 6 are in the range of an order of magnitude smaller. Maintaining a gain variance of 0.001 would require a scatterer position variance of 0.0001 in our wavelength-normalized units. For a 3GHz signal, the implied position accuracy is $0.01\lambda = 1$mm, a seemingly unrealizable accuracy when considering macroscopic objects over distances of tens or hundreds of meters.

Upon closer examination, however, we see that for our channel a scatterer could reside anywhere in a square region of $36 \times 10^4\lambda^2$. A 0.01λ accuracy implies a total of 36×10^8 possible positions. So, assuming uniformly random placement of each scatterer, about 32

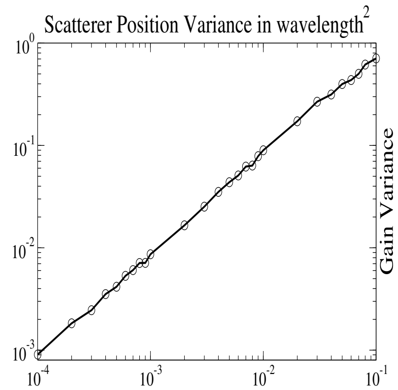

Figure 6:

bits of information would be required for localization per scatterer. Assuming completely stationary scatterers, how does this compare to the capacity of the interrogation channel given in equation (9)? Let us assume a simplistic sequence of probe vectors $\mathbf{p}(t) = \mathbf{e}_t$ where $\mathbf{e}_i$ is the cannonical unit vector. This decomposes the problem into a set of independent interrogation channels of the form

$$r_i(t) = g_{ti} + n_i(t) \tag{18}$$

one for each element of $\mathbf{G}$. Remembering that each element of the gain matrix is a zero mean Gaussian with unit variance, as is each $n_i(t)$, the mutual information between $r_i(t)$ and g_{ti} is exactly 0.5 bits (at the signal to noise ratio of 0dB implied by equation (18)), which implies a not unreasonable number of channel interrogations. So what at first seemed impossible – specifying scatterer position to high accuracy, does not now seem far-fetched even under the simplistic assumption that mean square gain matrix uncertainty is the proper figure of merit when speaking of MIMO channel capacity.

To elaborate on this last point, there is underlying structure of gain matrices with small variations in scatterer positions that was not accounted for in our broad brushstroke compare-the-variances approach. For example, from FIGURE 6 we see that positional variance of 0.001 implies a relatively large gain matrix entry variance of 0.01. However, in FIGURE 7 we plot 3 sets of gain points associated with this level of spatial perturbation along with the unperturbed set of gain points. It is apparent that the variation in the individual gain points is as large as given in FIGURE 6, however, there also seems to be significant structure relating the gain points of each channel instance. This structure is a result of the fact that small position changes do not strongly affect the $d_j(k)$ terms in the denominator of equation (15), while in contrast significant phase rotation can be introduced by small position variation [30].

These properties, imposed by the channel physics, could perhaps be used to further reduce the channel interrogation burden, but even when completely ignoring them, we have found that channel interrogation may not be that onerous from an information theoretic perspective. Of course, we have not yet taken into account the fact that scatterers move, often rapidly as measured in wavelengths. We address this concern by developing bounds on position uncertainty in the next section.

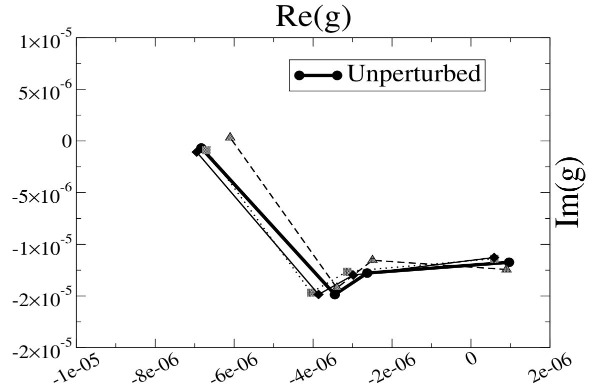

Figure 7: 2×2 MIMO Channels Subject to Scatterer Position Perturbation Variance of 0.001 Wavelength. The four gains for each channel instance are connected. Different channel instances are shown with different symbols.

4.2 Position Uncertainty

Fading channels are, by definition, time-varying and this time variation stems from physical movement of scatterers in the channel. In the previous section we argued that the data burden associated with specifying position sufficiently well is not too onerous. However, when scatterers are in high speed motion, the situation would seem to change.

Consider the 0.01λ quantization introduced in the previous section for a 3GHz system in an environment where scatterers moved at speeds of a meter per second (typical in an office environment). Our position quantization is one millimeter so one could expect 1000 significantly different channel instances per second. Multiplied by the 32 bits per scatterer in the previous example, the necessary implicit channel interrogation data rate would be a rather hefty 256kb/s. Of course, there would be correlations between successive channel states, so we might better use a rule of thumb estimate for channel coherence times of $0.4\lambda/v$ $[1, 32]$. In that case, we would observe an independent channel state once every 40ms for a necessary rate of ≥ 6.4kbps. In a setting with vehicles moving at tens of meters per second, the implicit rates would of course be commensurately higher.

However, this is precisely where the notion of positional *entropy rate* and the information processing theorem applied to FIGURE 4 becomes important. Although the channel gain matrix would appear to be highly volatile owing to scatterers in high speed motion, there is a chance that the channel could be tracked if the entropy rates associated with the moving parts are small enough so that $\mathcal{R}(\mathbf{G}) > \mathcal{H}(\mathbf{X})$. As an example, if the scatterer were following a trajectory which could be completely known given it's initial position, then $\mathcal{H}(\mathbf{G}) = \mathcal{H}(\mathbf{X}) = 0$ and we are left, essentially, with our original problem of estimating scatterer position though channel soundings at the relatively modest levels of the previous section.

However, if $\mathbf{X}$ is not completely predictable, given a quantization level Δx for spatial position, we can readily provide an upper bound $\mathcal{H}(\mathbf{X})$ using energy constraints. If we assume initial and final rest of an object, the minimum amount of energy to move a mass m a distance D by time τ is proportional to $m\left(\frac{D}{\tau}\right)^2$ [33–35]. So, suppose the position of an object is known at time $t = 0$ and we wait a time T before wishing to know its new position. We can ask, "what distribution p_k^* on its position $k\Delta x$ maximizes the position uncertainty?"

With an energy budget $\mathcal{E}$ we have the following optimization problem which we state in one spatial dimension, but which is easily extensible to three dimensions.

$$\{p_k^*\} = \arg \max_{\substack{p_k, \\ \sum_k p_k m \left(\frac{k\Delta x}{T}\right)^2 \leq \mathcal{E}}} \sum_k p_k \log \frac{1}{p_k} \tag{19}$$

whose solution is, not surprisingly, a discrete Gaussian-like distribution

$$p_k^* = \frac{1}{\sum_\ell e^{-\beta \ell^2}} e^{-\beta k^2} \tag{20}$$

where $\beta > 0$ is chosen to satisfy

$$\frac{1}{\sum_\ell e^{-\beta \ell^2}} \sum_k k^2 e^{-\beta k^2} = \frac{\mathcal{E}}{m} \left(\frac{T}{\Delta x}\right)^2 \tag{21}$$

The entropy of this distribution is then

$$H(p_k^*) = \log \left(\sum_\ell e^{-\beta \ell^2}\right) + \beta \frac{\mathcal{E}}{m} \left(\frac{T}{\Delta x}\right)^2 \tag{22}$$

Returning to our original concern, we consider scatterers (people with, say, 100kg mass) moving at a meter per second who move significantly ($\Delta x = 10^{-3}$m) every millisecond ($T = 10^{-3}$s). We then ask what uncertainty could we "inject" into this process under an energy constraint of 1J – which with our time step of 1ms is a significant power budget of 1kW! For this case, we have $\frac{\mathcal{E}}{m}\left(\frac{T}{\Delta x}\right)^2 = 0.01$ and we find $\beta = 5.28$ numerically. We can then evaluate the entropy as approximately 0.09 bits which translates into a 90bps entropy rate per scatterer for a total of $\mathcal{H}(\mathbf{G}) \leq 720$bps with our assumed 8 scatterers.

This constitutes a more than $\times 30$ reduction over per channel instance information rates, and about a $\times 10$ improvement over per-channel-coherence-time rates calculated previously.

5 Summary

With the previous exposition, we hope to have excited some interest in the possibility that using energy constraints on moving scattering elements could serve as a useful principle in the study of channel estimation in MIMO systems. For our simple examples, we have shown how modest constraints on the energy available to move an object in a stochastic fashion result in bounds on the *channel matrix process*. We have also argued that the fundamental problem of channel estimation is really about the comparison between this channel matrix process entropy rate with respect to an attainable *channel interrogation rate*. Elaborating on this view of MIMO channels is the heart of the proposed work.

Certainly there are many complaints that could be raised — details of the channel modeling, the anecdotal nature of the examples, details about channel interrogation and the like. However, the outlined approach to MIMO channel estimation seems potentially useful enough and also theoretically rich enough to provide a wealth of problems for study beyond those we have already raised in describing the approach.

For instance, supposing the details about channel entropy rates can be worked out in relation to channel interrogation rates. For cases where the interrogation rate is sufficiently high, the next obvious step is finding ways to form implicit channel models from channel interrogation. Simple (and not so simple) *a priori* channel models with known parametrizations may often be useful and will be studied, but acquiring the *channel generation model* blindly from channel soundings is an area worthy of exploration and falls in the general area of system identification [36].

Conversely, if the interrogation rate cannot support perfect channel knowledge, then we must ask how well we can estimate the channel – a sort of rate-distortion problem [31, 37] in this context. This particular issue touches on wireless secrecy. We might hope to know under what circumstances an evesdropper Eve can acquire sufficient information about the channel to enable decoding of messages sent between Alice and Bob. Some of the details will be in the specific channel models used, the resources available to Eve and the information above Eve's properties available to Alice and Bob, but the first metric applied should be interrogation rate vs. channel entropy rate. And obviously, this sort of problem can be recast in a setting where Eve, Alice and Bob are not adversaries, but rather, wish to avoid each other's transmissions – a mutual interference reduction problem.

As another example of potential research, consider that establishing an entropy rate for a system is in some sense a predictor. For channels which are erogodic in their underlying models – perhaps randomly arriving/departing scatterers with known characteristics, we might even attempt to apply various forms of empirical sequence prediction [38] using channel soundings as input, reminiscent of an approach which has been successfully applied in mobility tracking problems [39, 40]. We can even turn the problem completely around and seek to determine how one might artificially construct scattering environments which are completely untrackable or devise means which interfere optimally with channel tracking attempts by others under channel interrogration power constraints.

Hopefully, the above assortment of topics along with the exposition will be sufficient to convince even a highly skeptical reader of the intellectual interest and potential practical importance of the proposed work.

6 Project Management

6.1 Management

C. Rose will supervise graduate students, pursue publication in the relevant journals and conferences, as well as seek additional funding, possibly from other sources, to support an experimental complement to the proposed work. Below, the research topics discussed previously are arranged in (a tentative) sequence:

- Formulate simple analytically manageable scatter models of varying levels of detail and exercise them from the perspective of channel gain sensitivity to scatterer and channel interrogation sensor placement.

- Determine the amount of position/velocity information necessary to specify the channel to some (suitably defined) accuracy using some (suitably defined) metrics.

- Formulate an entropy rate of the underlying time-varying model for a variety of typical scatterer stochastic motion processes and upper bounds subject to mobility energy constraints.
- Compare the fundamental model entropy rate to the implicit information rates associated with standard channel interrogation methods
- Couch the channel estimation problem in terms of the underlying mobility energy constraints on the simple spherical scatterer model and determine what questions (probes) should be asked to most efficiently extract underlying model information.
- Consider the potential need for incorporating other more realistic physical scattering scenarios from the perspective of scattering center characterization, mapping and tracking.
- Provide answers to the question of when and how CSI can be efficiently extracted from interrogation of time-varying channels.

Throughout, special attention will be paid to the performance gains provided by good CSI. And as with all research, it is difficult to place things on a time line, so there could be considerable reordering, additions and deletions depending upon how the research evolves.

6.2 Impact

WINLAB's mission is the training of undergraduate and graduate students for the growing wireless industry, invention of new technology, adding to the archival wireless research literature and through WINLAB sponsors, technology transfer. Support for this proposed work will be used to further this mission. However, probably most important, knowing whether a MIMO channel can be tracked has an obvious and profound impact on wireless network provisioning and service offerings as wireless networks move to higher and higher capacities and densities in an increasingly wireless world. Furthermore, implicit in the ability to track MIMO channels is the ability to rapidly characterize such channels. If this research is successful, then the detailed knowledge of real MIMO channels which might be enabled by the proposed work cannot help but stimulate fundamentally new ways of thinking about wireless networks.

7 Results from Prior NSF Support

Christopher Rose: has served as PI and co-PI on a number of previous NSF grants; (PI) NCR-9206148 [41], CCR-98-14104 [42] and CCR-99-73012 [43]; (co-PI) NCR-9506505 [44], NCR-97-29863 [45], ITR/CCR-00-85986 [46], ITR/CCR 02-05362 [47], NeTS-0434854 [48] and NeTS-0435370 [49]. The work completed on these grants has addressed a broad range of problems associated with optimizing the use of radio resources in wireless communications systems. Call admission for wireless systems was studied in [50–53]. Fundamental algorithms for paging and registration of mobile nodes were established in [54–66]. Recent work has been focused on understanding the U-NII [42,67,68], opportunistic transmission methods and associated delivery protocols [69–71], and developing interference avoidance methods for a variety of communications problems [72–102] as well as non-standard communications models [33–35,103,104]. The work described in [35] is featured on the NSF *Discoveries* web page.

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