Minimizing the Average Cost of Paging and Registration: A Timer-Based Method

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ABSTRACT

Methods of balancing call registration and paging are developed in this paper. Given that the probability distribution on the user location as a function of time is either known or can be calculated, previous work shows the existence of lower bounds on the average cost of paging. Here these bounds are used in conjunction with a Poisson incoming-call arrival model to formulate the paging/registration optimization problem in terms of *timeout* parameters, τ_m; the maximum amount of time to wait before registering given the last known location was m. Timer-based methods, as opposed to location-based methods, do not require the user to record and process location information during the time between location updates. This feature might be desirable for minimizing mobile transceiver use during idle periods.

We then consider uniform motion processes where a spatial translation of starting location produces an identical spatial translation of the associated time-varying probability distribution. This leads to a universal timeout parameter τ which may be readily calculated. We study τ and the minimum cost of paging/registration for a simple model of user motion and compare our results to an earlier method of location-based paging/registration cost minimization.

1 Introduction

The whereabouts of a user in a mobile communication system must first be known in order to correctly route an incoming call. This user location is usually obtained via some combination of *paging* and *registration*. Paging is the process whereby the system issues polling signals in various locations and waits for user response. Registration is initiated by the user explicitly to notify the system of its location. Since bandwidth may be scarce at both the radio level and the underlying switching level, unnecessary paging over wide areas can impose an additional burden on the system [1, 2]. Requiring frequent registration would serve to reduce the range over which the system needs to search but could be equally onerous for high frequencies of registration. This inverse coupling of paging and registration costs forms the basis of the paging/registration optimization problem.

Currently, users register when they change *location areas* [3]. A location area is a group of locations (usually specified in terms of a radius) all of which are paged when an incoming call is directed to the user. Location areas are currently independent of the detailed characteristics of user motion. Thus, a user who registers in a metropolitan area at the start of the day and never leaves the workplace, might still be paged throughout the region. This suggests that using more specific location information might benefit system performance.

Under the rubric of location area management, a number of different schemes have been proposed for minimizing the cost of paging and registration; see for example [4]-[10]. However, optimal paging and registration, whether explicitly stated or not, is predicated on location estimation using some notion of user location probability. It therefore makes sense to explicitly separate the paging, registration, and probability distribution estimation problems. Three basic questions result:

1. Given a probability distribution, what is the least average amount of effort necessary (number of locations searched) to find a user? What is the effect of delay constraints?

2. Given a time-varying distribution known both by the user and the system, what are the optimal paging procedures based on information available at the mobile? I.e., location-based, timer-based or "state" based registration/paging.

3. How can these time-varying location probabilities be efficiently estimated based on measurement and/or models of user motion?

There are a number of approaches to the problem based on what information is available to whom (mobile unit and/or system). In this paper we develop timer-based methods of balancing call registration and paging using optimal paging algorithms developed elsewhere [11]. Timer-based methods, as opposed to location-based methods, do not require the user to record and process location information during the time between location updates. This feature might be desirable for minimizing mobile transceiver use during idle periods.

Specifically, we use a time-varying location probability distribution and previously developed analytic bounds on paging cost in conjunction with a Poisson page-arrival model, to formulate the paging/registration optimization problem in terms of a set of *timeout* parameters, $\{\tau_m\}$. Each τ_m is defined as the maximum amount of time to wait before registering given the last known location was m.[1] These $\{\tau_m\}$ could be calculated by the system and given to the user as necessary (at call termination or last registration). They might also be calculated by the user directly.

However, finding the optimal $\{\tau_m\}$ for spatially variant models of user motion is difficult in general owing to the subtle interaction of the registration process and conditional location probabilities. We therefore concentrate on the spatially-invariant case. Specifically, we illustrate our method on a time-varying Gaussian user location distribution which often arises as a result of isotropic random user motion [12]. We then compare our results to a simple location-based paging/registration minimization scheme [4].

2 Analysis

Let $p_{i|m}(t)$ be the probability distribution on a user residing in location i at time t given that its location was m at $t = 0$. It is assumed that this distribution, which depends on the underlying model of user motion, is either known or can be calculated. Implicit in the $i|m$ notation is the idea that future location i for $t > 0$ depends only upon the current location m at $t = 0$. However, this need not be the case. In a more general notation, m might be replaced by a location process state variable β. However, for the sake of clarity we retain the $i|m$ notation while recognizing that it may be extended whenever necessary.

We define a paging cost function, $F_m(t)$, as the cost of paging (average number of locations searched) given that the last known location (or location state) was m. $F_m(t)$ may be derived from arbitrary criteria and the conditional location probability distribution $p_{i|m}(t)$. We seek to minimize this cost.

[1] The simple case of memoryless motion was chosen for clarity. However, the framework can be easily extended to include any other type of ergodic user motion. Please see section 2 for further explanation.

It has previously been shown [11] that $F_m(t)$ is minimized by sequential search of each location in decreasing order of probability. Hence,

$$F_m(t) = \sum_{\ell=1}^{\infty} \ell p_{q_{\ell|m}}(t) \tag{1}$$

where q_ℓ is an ordering function such that $p_{q_{\ell|m}} \geq p_{q_{\ell+1|m}}$.

Notice then that the average number of polling events, and thus the average polling delay, is identical to the average number of locations polled. Although procedures exist whereby the average polling delay can be greatly reduced at the expense of modestly increased average number of locations searched [11], here we concentrate on the simpler case without delay constraints. This provides a lower bound on the average paging cost.

2.1 Average Paging Cost Without Registration

We assume a user's location is known to the system during the course of a conversation, just after a paging event and just after registration. We will concern ourselves with the time between the last known location and the next paging/registration event: the *roaming interval*. We do not consider intervals terminated by a call initiation since no additional cost is incurred. We will then assign costs to registration events and seek to minimize the time average cost of paging/registration.

To this end, assume the cost of registration is unity and the cost of paging is $\mathcal{P}$ per location. Since paging events are primarily generated by incoming calls, we assume that paging events form a Poisson point process of intensity λ_p independent of user motion. Since Poisson processes are memoryless, the time until the next paging event is described by the probability distribution $s_t(t) = \lambda_p \mathbf{e}^{-\lambda_p t}$ where t is referenced to the start of the roaming interval. The cumulative distribution function of $s_t()$ is $S_t(T) = 1 - \mathbf{e}^{-\lambda_p t}$.

If a user never registers, the expected cost of a roaming interval for which the last known location was m is then,

$$\mathcal{C}_m(\infty) = \mathcal{P} \int_0^\infty F_m(\sigma) s_t(\sigma) d\sigma \tag{2}$$

If κ_m is the probability of the last known location being m and we assume independent paging events, then the average cost of a roaming interval is,

$$\mathcal{C}(\infty) = \mathcal{P} \int_0^\infty \bar{F}(\sigma) s_t(\sigma) d\sigma \tag{3}$$

where

$$\bar{F}(\sigma) = \sum_m F_m(\sigma)\kappa_m$$

Since user motion and paging are assumed independent processes, $\kappa_m = \pi_m$, the steady state probability of residing in location m.

$\mathcal{C}$, however, gives no indication of the steady state expenditure rate. We therefore calculate the average cost per unit time of a roaming interval. Letting $d(m_i, i, \infty)$ be the duration of the i^{th} roaming interval and $c(m_i, i, \infty)$ its cost, we can define the average cost per unit time as

$$\eta(\infty) = \lim_{N\to\infty} \frac{\sum_{i=1}^{N} c(m_i, i, \infty)}{\sum_{i=1}^{N} d(m_i, i, \infty)} \tag{4}$$

which via the weak law of large numbers becomes

$$\eta(\infty) = \frac{\sum_m \mathcal{C}_m(\infty)\pi_m}{\sum_m \bar{d}(m, \infty)\pi_m} \tag{5}$$

where $\bar{d}$ is the average duration of a roaming interval. However, since the user never registers, the roaming interval is independent of location. Thus, the "null registration policy" average cost per unit time is,

$$\eta(\infty) = \lambda_p \sum_m \mathcal{C}_m(\infty)\pi_m \tag{6}$$

2.2 Average Cost of Joint Registration and Paging

Suppose that the last known user location is m at $t = 0$. The user is assumed to register at time $t = \tau_m$ if a paging event does not occur first. Thus, the probability of the roaming interval being terminated by a paging event is $S_t(\tau_m)$, and $1 - S_t(\tau_m)$ by a registration. The expected registration cost is then $1 - \mathcal{S}_t(\tau_m)$ so that the total cost is

$$\mathcal{C}_m(\tau_m) = \mathcal{P}\int_0^{\tau_m} F_m(\sigma)s_t(\sigma)d\sigma + (1 - S_t(\tau_m)) \tag{7}$$

The probability distribution on the duration $d(m)$ of a roaming interval started in location m is

$$p_{d(m)}(d) = s_t(d) + \delta(d - \tau_m)(1 - S_t(\tau_m)) \tag{8}$$

for $0 \le d \le \tau_m$ where $\delta()$ is the unit impulse function.

Letting $d(m_i, i, \tau_{m_i})$ be the duration of the i^{th} roaming interval and $c(m_i, i, \tau_{m_i})$

its cost, we can define the average paging/registration cost per unit time as

$$\eta(\vec{\tau}) = \lim_{N\to\infty} \frac{\sum_{i=1}^{N} c(m_i, i, \tau_{m_i})}{\sum_{i=1}^{N} d(m_i, i, \tau_{m_i})} \tag{9}$$

where $\vec{\tau}$ is a vector composed of the τ_m. Via the weak law of large numbers we have

$$\eta(\vec{\tau}) = \sum_m \mathcal{C}(m)\kappa_m \Big/ \sum_m \bar{d}(m, \tau_m)\kappa_m \tag{10}$$

Notice that here κ_m is not necessarily the steady state probability of being in location m. This difference is caused by the dependence of κ_m on the registration process which is itself dependent on user location. Thus, for each choice of the $\{\tau_m\}$, the associated κ_m must be calculated as well. This coupling makes it difficult to optimize the $\{\tau_m\}$ in general. For example, say $\tau_1 = 0$ and all other τ_m are infinite. In location 1 we would have an infinite number of registrations. Thus, regardless of the steady state location distribution π_m, we would have $\kappa_1 = 1$.

Nonetheless, we may formally determine the κ_m in terms of the τ_m by first defining a probability distribution $q_{l|m}$, the probability that the next known location will be l given the last known location was m.[2] Since we assume the probability distribution $p_{l|m}(t)$ is known we can write

$$q_{l|m} = \int_0^{\tau_m} p_{l|m}(t) s_t(t) dt + (1 - \mathcal{S}_t(\tau_m)) p_{l|m}(\tau_m) \tag{11}$$

owing to the assumed independence of the paging and motion processes.

Now we note that $q_{l|m}$ describes a Markov process on the last known location and that the κ_m form the stationary distribution for this process. It is therefore possible, using suitable numerical methods, to obtain the κ_m as functions of the τ_m, and thereby find the optimal set $\{\tau_m\}$ which minimizes $\eta(\vec{\tau})$

To gain analytic insight, however, in what follows we consider only spatially invariant user motion processes with identical distributions on location trajectories from any starting point; i.e., the κ_m are effectively uniform.

[2]Once again, it is important to note that general location process state variables χ and β could be substituted for the locations l and m. Thus, the motion process need not necessarily be Markov on the current location m.

3 Cost Minimization

Here we assume that the motion process is independent of the starting location; i.e. $F_m(t) = F(t)$. A number of motion models, such as unconstrained random walks or walks on finite rings, spheres or tori are independent of starting state in that the probability distribution derived from starting in any one state is simply a spatial translation of that derived from starting in another state. Since the registration policy for each last known location should be identical under these conditions, we have $\tau_i = \tau_j = \tau$ so that

$$\mathcal{C} = \mathcal{P} \int_0^\tau F(\sigma) s_t(\sigma) d\sigma + (1 - S_t(\tau)) \tag{12}$$

and

$$\bar{d}(\tau) = \int_0^\tau \sigma s_t(\sigma) d\sigma + \tau(1 - S_t(\tau)) \tag{13}$$

For a Poisson paging process we have

$$\eta(\tau) = \frac{\lambda_p}{1 - \mathbf{e}^{-\lambda_p \tau}} \left(\mathcal{P} \int_0^\tau F(\sigma) \lambda_p \mathbf{e}^{-\lambda_p \sigma} d\sigma + \mathbf{e}^{-\lambda_p \tau} \right) \tag{14}$$

We then seek the τ which minimizes equation (14). Differentiating both sides and setting the result to zero shows that extremal points occur as $\tau \to \infty$ and when

$$\eta(\tau) = \lambda_p(\mathcal{P}F(\tau) - 1) \tag{15}$$

Equations (14) and (15) allow some general conclusions to be drawn. Since $F(\tau) > 0$ and $\eta(\tau) > 0$, if

$$\mathcal{P} \max_\tau F(\tau) < 1$$

then it is best never to register since the cost of paging is always smaller than that of registering. Conversely, if $\mathcal{P}F(\tau)$ increases without bound in τ at any rate such that

$$F(\tau) \leq G\mathbf{e}^{\sigma\tau}$$

for some suitably chosen $G > 0$ and $\sigma < \lambda_p$, then because η must be bounded, some optimum $\tau > 0$ always exists for which the cost $\eta(\tau)$ is minimized. Finally, if $F(\tau)$ increases rapidly with τ then equations (14) and (15) suggest that frequent registration (small τ) is optimal.

4 Application to a Simple Model of User Motion

4.1 Optimum τ and $\eta(\tau)$ for Isotropic Diffusive Motion

A number of motion models which are specified in terms of independent increments result in Gaussian distributions on location probability [12], [13]. The simplist is diffusive motion on a line which has probability density,

$$p_g(x,\tau) = \frac{1}{\sqrt{\pi \mathcal{D} \tau}} \mathbf{e}^{\frac{-(x-vt)^2}{\mathcal{D}\tau}} \tag{16}$$

where x is the location variable, $\mathcal{D}$ is the diffusion coefficient (units of length^2/time), and v is the mean velocity. Two-dimensional motion is considered in Appendix A.

To obtain minimum average paging we must search each location in decreasing order of probability [11]. The most likely location is the mean of the distribution with symmetric equally likely locations to either side of the mean. Thus, assuming that locations are quantized in steps of δ centered about the mean vt we have,

$$F(\tau,\delta) = p_x(0,\tau) + \sum_{n=1}^{\infty} (4n+1) p_x(n,\tau) \tag{17}$$

where

$$p_x(n,\tau) = \int_{(n-\frac{1}{2})\delta+vt}^{(n+\frac{1}{2})\delta+vt} p_g(x,\tau)dx = \int_{(n-\frac{1}{2})\delta}^{(n+\frac{1}{2})\delta} \frac{1}{\sqrt{\pi \mathcal{D}\tau}} \mathbf{e}^{\frac{-x^2}{\mathcal{D}\tau}} dx$$

Notice then that $F(\tau,\delta)$ is independent of the mean velocity; i.e., $F(\tau,\delta)$ reflects positional *uncertainty* rather than the rate of change in absolute position.

A useful simplification can be had if we define $\mathcal{F}(\tau,\delta) = F(\tau,\delta)\delta$. Letting $\chi = n\delta$ and taking the limit of $\delta \to 0$ we find

$$\mathcal{F}(\tau) = \int_0^{\infty} \frac{4\chi}{\sqrt{\pi \mathcal{D}\tau}} \mathbf{e}^{-\chi^2/\mathcal{D}\tau} d\chi \tag{18}$$

The integral of equation (18) may be simplified to,

$$\mathcal{F}(\tau) = \frac{2\mathcal{D}\tau}{\sqrt{\pi \mathcal{D}\tau}} = \sqrt{\frac{4\tau\mathcal{D}}{\pi}} \tag{19}$$

Notice that although $\mathcal{F}(\tau)$ is unbounded as $\tau \to \infty$, we can find a $G > 0$ and $\sigma < \lambda_p$ such that,

$$\mathcal{F}(\tau) \leq G\mathbf{e}^{\sigma\tau}$$

Thus, an optimal $\tau > 0$ which minimizes η exists. Also note that $\mathcal{F}(\tau)$ has units of distance.

We then have

$$\eta(\tau) = \frac{\lambda_p}{1-\mathbf{e}^{-\lambda_p\tau}}\left(\mathcal{P}\sqrt{\frac{4\mathcal{D}}{\pi}}\int_0^\tau \sqrt{z}\mathbf{e}^{-\lambda_p z}\lambda_p dz + \mathbf{e}^{-\lambda_p\tau}\right) \tag{20}$$

Substituting $t = \tau\lambda_p$ (and $\omega = z\lambda_p$) while defining a mobility index $\rho = \mathcal{D}/\lambda_p$ produces,[3]

$$\eta(t) = \frac{\lambda_p}{1-\mathbf{e}^{-t}}\left(2\mathcal{P}\rho^{1/2}\int_0^t \sqrt{\frac{\omega}{\pi}}\mathbf{e}^{-\omega}d\omega + \mathbf{e}^{-t}\right) \tag{21}$$

Using the identity [14] [15]

$$\int \sqrt{\omega}\mathbf{e}^{-\omega}d\omega = \frac{1}{2}\sqrt{\pi}\mathrm{erf}(\sqrt{\omega}) - \sqrt{\omega}\mathbf{e}^{-\omega}$$

we obtain

$$\eta(t) = \frac{\lambda_p}{1-\mathbf{e}^{-t}}\left(\rho^{1/2}\mathcal{P}[\mathrm{erf}(\sqrt{t}) - 2\sqrt{\frac{t}{\pi}}\mathbf{e}^{-t}] + \mathbf{e}^{-t}\right) \tag{22}$$

Observe the monotonically increasing cost associated with increasing mobility index ρ.

It is worthwhile to note that similar expressions can be obtained for motion in two dimensions. The primary difference is the growth rate of $\mathcal{F}(\tau)$ in τ. For two dimensions $\mathcal{F}(\tau) \propto \tau$ which leads to paging component in η which varies linearly in ρ (see Appendix A for details). However, here we concentrate on linear motion for ease of comparison to previous methods.

For clarity, we hereafter assume either that $\lambda_p = 1$ or that η is measured in cost per average page interarrival time, $1/\lambda_p$. Regardless of how we choose to think about it, the end result is that the multiplicative factor of λ_p disappears from equation (22). A family of curves for $\eta(t)$ parametrized in ρ is shown for $\mathcal{P} = 0.1$ in FIGURE 1. Pronounced optimum t^* are readily apparent for higher mobility indices. We also plot the minimum η as a function of $\sqrt{\rho}$ for $\mathcal{P} = 0.1$ and 0.01 in FIGURE 2.

4.2 Comparison to Simple Location-Based Updating

Consider the currently popular location-based update scheme where a user is assigned a location area and registers the first time it breaches the area boundary. One might then consider sizing the location area optimally as a function of ρ the user mobility

[3]Notice that here the mobility index is the ratio of the diffusion coefficient to page arrival rate; an intuitively reasonable quantity.

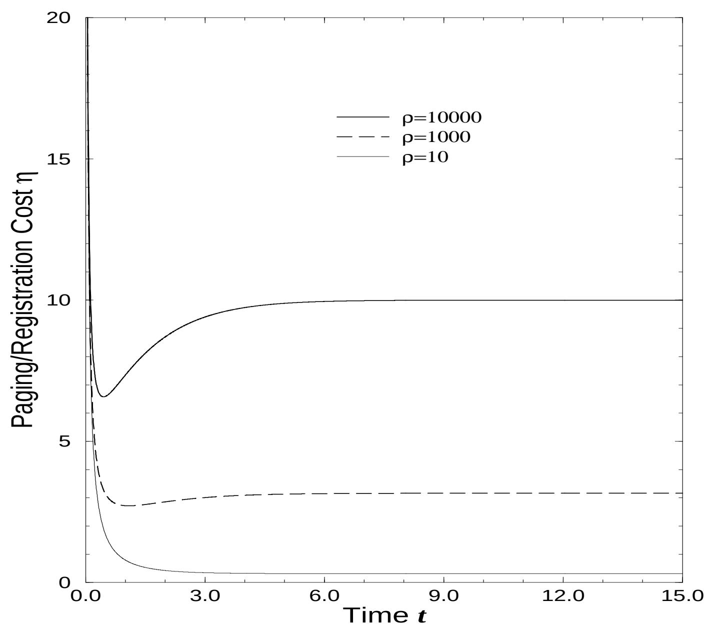

Figure 1: *Mean cost per normalized unit time $\eta(t)$ for diffusive motion process with paging cost $\mathcal{P} = 0.1$ and various mobility indices ρ.*

index [4].

The optimization problem then becomes one of finding the interval size which minimizes the paging/registration cost per unit time. The location area is assumed to be an interval symmetric about the last known location. In keeping with previous work, we will assume the average paging cost per unit time is proportional to the product of the interval size and the page arrival rate; $2\lambda_p\mathcal{I}$. Normalizing by λ_p (or setting $\lambda_p = 1$) we have,

$$\mu(\mathcal{I}) = 2\mathcal{P}\mathcal{I} + 1/T(\mathcal{I}, \rho, \nu) \tag{23}$$

where as before ρ is the mobility index, ν is the mean velocity and $T(\mathcal{I}, \rho, \nu)$ is the mean time to first boundary crossing at $\pm\mathcal{I}$ both referenced to the mean page interarrival time, $1/\lambda_p$.

Following [13] (also see Appendix B), we may find $T(\mathcal{I}, \rho, \nu)$ as,

$$T(\mathcal{I}, \rho, \nu) = \begin{cases} 2\mathcal{I}^2/\rho & \nu = 0 \\ \dfrac{\mathcal{I}}{\nu}\tanh(\dfrac{2\nu\mathcal{I}}{\rho}) & \nu \neq 0 \end{cases} \tag{24}$$

For $\nu = 0$, the optimum location area (interval) size is readily calculated as $\mathcal{I}^* = \sqrt[3]{\frac{\rho}{2\mathcal{P}}}$.

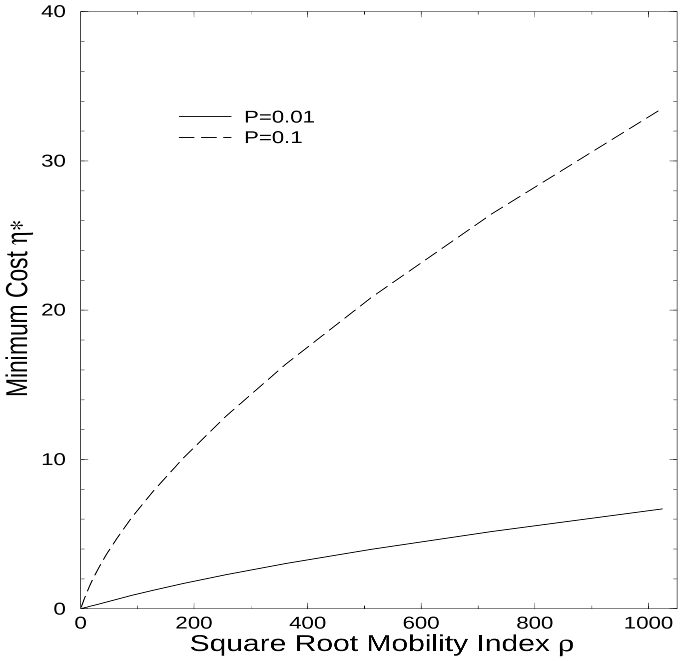

Figure 2: *Minimum mean cost per normalized unit time* η_{min} *versus the square root of the mobility index* ρ *for a diffusive motion process with paging costs* $\mathcal{P} = 0.1$ *and* 0.01.

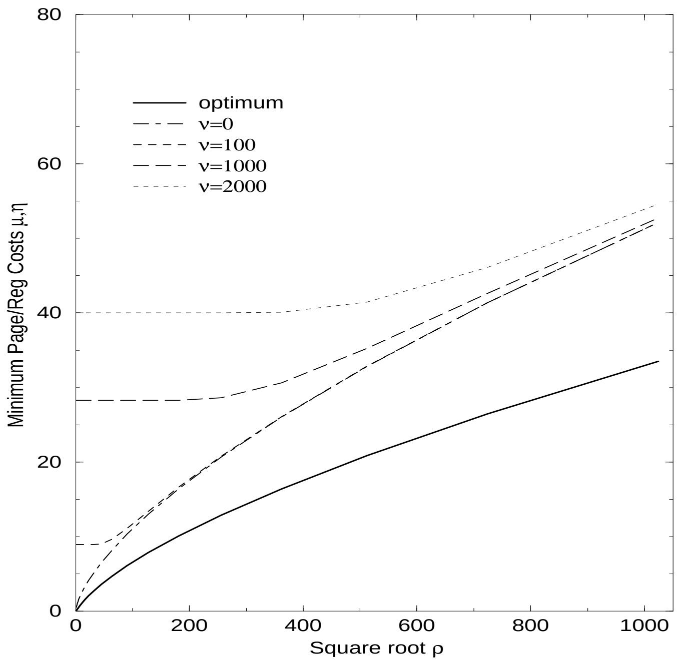

Figure 3: *Cost comparison of a simple location-based paging/registration scheme and the optimal timer-based methods developed here. Drift velocity ν as shown. Notice that the optimum method is independent of ν.*

This leads to

$$\mu(\mathcal{I}^*) = 3\sqrt[3]{\frac{\rho \mathcal{P}^2}{2}} \tag{25}$$

For $\nu \neq 0$, $\mu(\mathcal{I}^*)$ must be calculated numerically.

In FIGURE 3 we compare the optimized mean cost for this location-based method to that of the optimum timer-based method as a function of mobility index ρ and various ν. We set the relative paging cost $\mathcal{P} = 0.1$. The optimum timer-based paging/registration method both underbounds and greatly outperforms the previous method especially for larger ν.

5 Discussion & Conclusions

Given any spatially invariant model of user motion, the resultant time-varying location probability distribution, a paging arrival process and the relative cost of paging, we showed how to calculate an optimum time-out parameter, τ which minimizes the average cost of paging/registration. Thus, if an incoming call or a new call initiation has not occurred within τ seconds of the last registration or call termination, the user should register its location with the system. The parameter τ should be relatively easy to calculate for a wide range of spatially invariant motion models after calculation of the associated expected paging cost $F(\tau)$.

For the case where motion characteristics vary as a function of location, we showed that the optimization problem is in general difficult. However, a theoretical framework through which suitable numerical methods could be applied was also provided.

We then solved the optimal timer-based paging/registration problem for a simple model of user motion in one and two dimensions. and found that timer-based minimum paging/registration cost is dependent on location *uncertainty* rather than mean user velocity. This point is theoretically important since it is natural to think of high mobility users as those which move rapidly. For example, an automobile moving at a steady 100 kph in a known direction has a mobility index of $\rho = 0$ for the paging/registration problem; i.e., there is no uncertainty in the location given the velocity, time and starting point.

Owing to the difficulty of deriving general analytic expressions for mean first passage time under two dimensional isotropic diffusion we restricted our study to the one-dimensional case. We found that our method performed substantially better than a fixed location area-based procedure. The general form of the optimization equations, results from appendix A and heuristic bounds on the mean first passage time in two dimensions lead us to believe that similar comparative results can be expected for two dimensions. Regardless, the assertion that a timer-based method performed better than a location-based method is misleading since it is not necessarily true that all location-based registration schemes necessarily perform more poorly than timer-based schemes.

For example, optimal paging techniques could be applied to the location-based scheme thereby reducing the effective paging cost $\mathcal{P}$ by at least half. For $\nu = 0$, this modification makes the location-based scheme perform almost as well as the timer-based scheme. In addition, if paging area boundaries are allowed to change during the roaming interval, then current research indicates that the strong dependence on ν can be removed [16].

In both this paper and in [11], the problem of how $p_{i|m}(t)$ may be derived or measured is left open. Future work should consider the issue of location process characterization and prediction. Recent results in optimal prediction theory based on empirical sequences [17] may prove useful in this regard.

It is also possible to formulate the optimal paging/registration problem in terms of decision theory. In particular, one may assume the mobile terminal knows its location as well as the cost to be incurred by a paging event at time t. In as much as this information characterizes the state of the system, optimal decision methods such as Markovian Decision Theory [18, 19] can be brought to bear. Furthermore, this problem might be generalized to include whatever state information is available in a practical sense such as time of day, system geography or anything else easily obtainable. Methods which use both location and time information are currently under study [20].

A natural extension of this line of thought leads to the notion that the probability distribution used for optimal paging by the system depends upon the registration scheme used by the mobile. Thus, one could envision a treatment of the problem similar to cooperative game theory [21, 22] whereby the registration policy is iteratively refined until a fixed point in the policy space is identified. Such an approach, if successful would allow the calculation of absolute lower bounds on paging/registration costs.

A $\mathcal{F}(\tau)$ and $\eta(\tau)$ for Two-Dimensional Motion

For two dimensions, the probability density for isotropic diffusion with drift is

$$p_g(x, y, \tau) = \frac{1}{\pi \mathcal{D}_2 \tau} \mathbf{e}^{-\frac{(x - v_x \tau)^2 + (y - v_y \tau)^2}{\mathcal{D}_2 \tau}} \tag{26}$$

Assume that the regions to be paged are annuli of equal area A. Thus the annuli, centered about the mean $(v_x, v_y)\tau$, will be searched from smallest to largest radius. The first annulus is a disk of radius $\sqrt{A/\pi}$. The second annulus has outer radius r_2 which must satisfy $r_2^2 - r_1^2 = A/\pi$. Thus, $r_2 = \sqrt{2A/\pi}$. Proceeding inductively we obtain $r_n = \sqrt{nA/\pi}$.

After rewriting the probability distribution in polar coordinates, $F_2(\tau)$ is then

$$F_2(\tau) = \sum_{n=1}^{\infty} n \int_{\sqrt{\frac{(n-1)A}{\pi}}}^{\sqrt{\frac{nA}{\pi}}} \frac{2r}{\mathcal{D}_2 \tau} \mathbf{e}^{-\frac{r^2}{\mathcal{D}_2 \tau}} dr = \frac{1}{1 - \mathbf{e}^{-A/\pi \mathcal{D}_2 \tau}} \tag{27}$$

Defining $\mathcal{F}_2(\tau) = AF_2(\tau)$ and taking $A \to 0$ we obtain

$$\mathcal{F}_2(\tau) = \pi \mathcal{D}_2 \tau \tag{28}$$

which increases without bound in τ but which satisfies

$$\mathcal{F}_2(\tau) \leq G\mathbf{e}^{\sigma\tau}$$

for suitable $G > 0$ and $\sigma < \lambda_p$. Thus, the associated paging/registration cost $\eta_2(\tau)$ has a minimum for some $\tau > 0$ (see text).

The expression for η_2 analogous to equation (22) is then

$$\eta_2(t) = \frac{\lambda_p}{1 - \mathbf{e}^{-t}} \left(\mathcal{P}\rho \int_0^t \omega\pi \mathbf{e}^{-\omega} d\omega + \mathbf{e}^{-t} \right) = \lambda_p \mathcal{P}\rho\pi + \frac{\lambda_p \mathbf{e}^{-t}}{1 - \mathbf{e}^{-t}} (1 - \mathcal{P}\rho\pi t) \tag{29}$$

where $\rho = \mathcal{D}/\lambda_p$ is defined as the mobility index. Thus, the paging cost contained in η_2 is linear in ρ. for two-dimensional motion. This result implies that as a general rule, given a mobility index ρ, the timer value which minimizes η_2 will be smaller than that of the corresponding one-dimensional motion; i.e., the paging cost grows more rapidly with τ in two dimensions than in one dimension and thereby requires more frequent registration.

B Mean First Passage Time for Random Walks

The following derivation is an adaptation and extension of methods used by [13].

Consider a particle at position integer position $0 \leq X \leq R$ at time $t = 0$. At each time step, the particle moves one unit to the right with probability p and left with probability $1 - p$. Using a recursive method [13], the mean number of steps before the particle touches boundaries at 0 or R for the first time is,

$$D(X, p = 1/2) = X(R - X) \tag{30}$$

and

$$D(X, p \neq 1/2) = \left[R \frac{1 - \left(\frac{1-p}{p}\right)^X}{1 - \left(\frac{1-p}{p}\right)^R} - X \right] \frac{1}{2p - 1} \tag{31}$$

Now let the steps be of size Δx and let the steps occur at integer multiples of Δt. Define $t = \Delta t T$ where T is the number of steps. We can define the particle position $x(t)$ as

$$x(t) = x(0) + \sum_{i=1}^{T} \Delta x_i$$

Assuming that each step is independent and $x(0) = 0$ we have approximately

$$p_{x(t)}(x(t)) \approx N[t \frac{\Delta x}{\Delta t}(2p - 1), t \frac{\Delta x^2}{\Delta t} 4p(1 - p)]$$

I.e., the distribution is approaches normal for a large number of independent steps, T. Following convention [12], we define diffusion constant $\mathcal{D}$ and mean velocity v as

$$\frac{\mathcal{D}}{2} = \frac{\Delta x^2}{\Delta t} 4p(1 - p)$$

and

$$v = \frac{\Delta x}{\Delta t}(2p - 1)$$

respectively. We further define $X = x/\Delta x$, $R = r/\Delta x$ and $d(x) = D(X)\Delta t$.

For zero mean velocity we have $p = 1/2$. Thus, we obtain

$$d(x, v = 0) = \frac{2x(r - x)}{\mathcal{D}} \tag{32}$$

For $p \neq 1/2$ we have

$$d(x, v \neq 0) = \frac{1}{v}\left[r\frac{1-\left(\frac{1-p}{p}\right)^{x/\Delta x}}{1-\left(\frac{1-p}{p}\right)^{r/\Delta x}} - x\right] \tag{33}$$

but we desire an expression in terms of $\mathcal{D}$ and v rather than Δx and p.

The definition for v yields

$$p = (v\Delta t/\Delta x + 1)/2$$

and combined with the definition of $\mathcal{D}$ yields

$$\Delta x = \sqrt{v^2\Delta t^2 + \mathcal{D}\Delta t/2} \approx \sqrt{\mathcal{D}\Delta t/2}$$

for small Δt. From this we obtain

$$p \approx \frac{1}{2} + \frac{v}{2}\sqrt{\frac{2\Delta t}{\mathcal{D}}}$$

Substitution of these results into equation (33), letting $\Delta t \to 0$ and utilizing the identity

$$\lim_{n\to\infty}(1 - a/n)^{bn} = \mathbf{e}^{-ab}$$

with $a = \frac{2v}{\sqrt{D/2}}$, $b = \frac{x}{\sqrt{D/2}}$ and $n = \frac{1}{\sqrt{\Delta t}}$ yields

$$d(x, v \neq 0) = \frac{1}{v}\left[r\frac{1-\mathbf{e}^{-\frac{4v}{\mathcal{D}}x}}{1-\mathbf{e}^{-\frac{4v}{\mathcal{D}}r}} - x\right] \tag{34}$$

For the special case of $x = r/2$ we can simplify equation (34) to obtain

$$d(r/2, v \neq 0) = \frac{r}{v}\left[\frac{1-\mathbf{e}^{-\frac{vr}{\mathcal{D}}}}{(1-\mathbf{e}^{-\frac{2vr}{\mathcal{D}}})(1+\mathbf{e}^{-\frac{2vr}{\mathcal{D}}})} - \frac{1}{2}\right] = \frac{r}{2v}\tanh(\frac{vr}{\mathcal{D}}) \tag{35}$$

Derivation of corresponding results in two dimensions is difficult [13, 23].

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