INTERMEDIATION AND EXCLUSIVE REPRESENTATION IN FINANCIAL NETWORKS

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ABSTRACT. This paper develops a theory of financial intermediation. Consider an environment with repeated financial interactions in which strategic default is possible. We show that if interactions between any lender and borrower are infrequent and if market participants have incomplete knowledge of the patterns of interactions in the market, then all investments must be intermediated. Moreover, each intermediary must exclusively represent many lenders in their interactions with a given borrower, so that she can "punish" the borrower severely for any strategic default by eliminating the borrower's access to many future loans. To this end, we develop a model of financial networks that are shaped by exogenous forces as well as by lenders' decisions and new tools to study the ability of market participants to learn about the structure of the financial network. We then characterize networks that are robust – networks that can be sustained in equilibrium given (almost) any belief that is consistent with agents' knowledge of the network structure. Our characterization sheds light on the complementarity and substitutability of self-finance clauses, the use of collateral, and intermediation; and suggests also that the riskiest and safest assets will be traded using full collateral, whereas the intermediately risky assets will be traded by intermediaries and without full collateral. The effect of macroeconomic conditions and the presence of credit bureaus on the patterns of intermediation are also studied.(JEL: C73, D42, D83, D85, D86, G20, L14)

Key words: Financial intermediation, strategic default, networks, repeated games, knowledge of network structure, moral hazard, social capital.

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Contents

1. Introduction

This paper develops a theory of financial intermediation based on the resolution of incentives problems via repeated interactions. Consider a potential lender (e.g. investor) and a borrower (e.g. entrepreneur) who needs to raise capital for a risky project. Even if both parties know the expected returns of the project and observe its outcome, limited liability combined with lack of verifiability of the realization of the risky investment leave open the possibility of strategic default.[1] If the frequency with which the lender has liquidity (when the borrower needs liquidity) is low, then strategic default by the borrower cannot be deterred by a threat of losing access to future funds from the lender. However, a financial intermediary (e.g. an investment bank) who exclusively represents a large pool of lenders in their transactions with a borrower can still enforce repayment by the borrower. The intermediary can do that by threatening to eliminate the borrower's access to future funds from many lenders.

Clearly, we are not the first to study game theoretic foundations for the enforcement of informal contracts. The literature on community enforcement offers two enforcement mechanisms to explain the prevalence of informal contracts in the presence of incentives problems. One mechanism is *ostracism.*[2] Ostracizing a borrower requires coordination. In some markets coordination is achieved by tight social groups, i.e. family or an ethnic group (e.g. Greif 1993 and Munshi 2011). When a market is not dominated by social groups, coordination requires common observations and common knowledge of the patterns of interactions between agents. A second mechanism suggested in the literature is *contagion* – any agent who observes a default reacts by defaulting (if a borrower) or by avoiding the provision of liquidity (if lender), independent of the identity of their trading partner.[3] Contagion is hard to motivate in large markets (Kandori 1992, Ellison 1994), and requires implicit coordination between agents in order to provide the incentives to spread 'bad behavior' to the entire population.

In this paper we propose a third mechanism – *intermediation.* If each of a group of lenders agrees to invest with a given borrower only via a given intermediary, the intermediary can single-handedly 'cut off' a defaulting borrower's access to liquidity from a large group of lenders.

[1] In a strategic default, a firm defaults because managers divert cash to themselves (see also Bolton and Scharfstein 1990, Bolton and Scharfstein 1996, and Hart and Moore 1998). Much of the finance literature on intermediation assumes that the strategic nature of a default may not be verifiable in court, or that its verification is costly. This is often done by assuming that the realizations of firm level risky investments are (in the language of Grossman and Hart 1986) observable but not verifiable (e.g. see Bolton and Scharfstein 1996 and Babus 2010). One motivating example is the common covenant that the firm's working capital will not fall below some minimum, unless "necessary for expansion of inventory" (see Smith and Warner 1979 and Diamond 1984). While a lender may have insights as to whether the expansion of inventory was necessary, it might be much more cumbersome to prove it in court.

[2] See Greif (1993) and in the context of social networks also Babus (2010), Ali and Miller (2012b), and Fainmesser and Goldberg (2011).

[3] See Kandori (1992), Ellison (1994), and in the context of social networks also Haag and Lagunoff (2006), Ali and Miller (2012a), Mihm, Toth, and Lang (2009), Lippert and Spagnolo (2011), and Nava and Piccione (2011).

Our analysis suggests that well positioned intermediaries can enforce repayment in environments in which ostracism and contagion cannot.

Previous theories of financial intermediation are based on cost advantage for the intermediary. Schumpeter (1939), Leland and Pyle (1977), Chan (1983), Diamond (1984), and Babus (2010) assign a delegated monitoring role to intermediaries.[4] Other theories of intermediation (not necessarily financial) abstract from the resolution of incentives problems that intermediation provides, and focus on the role of intermediaries in overcoming market frictions in models of search and bargaining (see also Rubinstein and Wolinsky 1987, and Duffie, Gârleanu, and Pedersen 2005). We offer a new and complementary explanation that focuses on the resolution of incentives problems, yet does not require assuming exogenous cost advantages, economies of scale, or trade frictions.[5]

A novel feature of our model is that agents are not assumed to observe the network structure directly; agents observe their own financial interactions, and their knowledge of the network is derived as an upper bound on what they would be able to learn about the network structure based on their observations in many such interactions. The idea that agents do not observe the network structure directly, but rather infer the network structure from their observations during their own interactions is reasonable given that the network is not a physical object, but rather a collection of relationships that generate the activity in the economy.

Given that agents receive information only on parts of the network that affect their own financial interactions, some forms of community enforcement are infeasible. If a borrower b strategically defaults, his link with the lender or intermediary, say k, who provided him with the liquidity is lost. However, for any additional intermediary or lender, say j, to disconnect her link to the borrower, two conditions need be fulfilled: [1] j observes the default (or the elimination of the link between k and b); and [2] given her observations and beliefs, j has the incentives to eliminate her link to b rather than "pretend" not to have observed the default (or the elimination of the link between k and b). For example, j may prefer to "cover-up" b's default if j believes that other lenders or intermediaries did not observe the default (or the elimination of the link between k and b) and that b has sufficient incentives not to default on j as long as no additional links are eliminated.

The main result of the paper offers a complete characterization of the set of networks that are *robust* – networks that can be sustained in pure strategy perfect Bayesian equilibria of the

[4]Babus (2010) offers a model of repeated games in networks in which monitoring is costless along existing links, and the cost of monitoring is captured by a fixed cost of maintaining each link.

[5]A related explanation is suggested by Biglaiser and Friedman (1994). In their paper, middlemen are assumed to increase the speed of diffusion of information on sellers' defections (i.e. quality reductions), thus deterring sellers from defecting. However, their focus is different than ours. They take as given the process of information diffusion and the effect of information diffusion on demand, and derive the minimal price premium that is sufficient to incentivize sellers to produce high quality. In contrast, our focus is on the effect of intermediation on demand in a strategic setup.

infinitely repeated game given any belief from a large set of beliefs that we consider. We show that there exists a mapping from the parameters of the model to a positive integer m such that in robust networks any active intermediary is an *m-local monopoly* – for every borrower who the intermediary is connected to, she is also connected to at least m lenders who are not connected to the borrower in any other way, either directly or via another intermediary. That is, any intermediary exclusively represents at least m lenders in transactions with any borrower that she is connected to. Figure 1.1 demonstrates the notion of local monopolism.

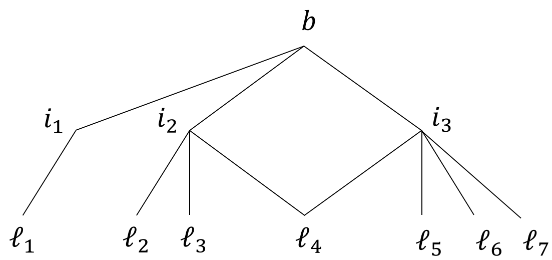

FIGURE 1.1. Local monopolism. Intermediary i_1 is a 1-local monopoly, i_2 is k-local monopoly for any $k \leq 2$, and i_3 is k-local monopoly for any $k \leq 3$.

We also show that if the parameters of the model are such that $m > 1$, then in all robust networks *all* investments are intermediated. This explains the presence of intermediaries even in markets in which there is no exogenous cost advantage to intermediation. The requirement that an intermediary provide each of the borrowers connected to her with unique access to at least m lenders, highlights that the important factor is not the absolute size of an intermediary, or the overall number of investments that she intermediates, but rather the exclusivity over a sufficient number of *investment paths*. Such exclusivity can be achieved by a large intermediary, but it can also be achieved by an intermediary who specializes and focuses on a small (but not too small) number of borrowers and lenders that cannot transact otherwise. For example, an intermediary can focus on local businesses, or can provide a connection between otherwise disconnected communities. In that sense, our results provide new insights that are related to the discussion of the optimal size of financial intermediaries.

By relating m to the parameters of the model, we are able to show that the minimal level of monopoly power that an intermediary is required to hold (as captured by m) decreases in the frequency of arrival of investment opportunities and the expected return on investment, and increases in the borrowers discount rate and in the return on capital demanded by lenders.

From a macroeconomic perspective, the model predicts that in times of economic booms (high frequency of arrival of investment opportunities and high expected returns on investments) there is room for a large number of intermediaries and more competitive markets in which no single intermediary has significant market power (as captured by m). On the other hand, in times of economic downturns, especially ones that are triggered by liquidity crunches, the model predicts a smaller number of intermediaries, each with significant market power.

We also find that requiring that a borrower self-finance a positive fraction of the investment opportunity reduces the monopoly power required by intermediaries in order to enforce repayment, and that the same is true for partial collateral and for the presence of bankruptcy laws. On the other hand, we find that introducing the possibility of pledging full collateral increases the monopoly power that intermediaries are required to have in order to enforce repayment of uncollateralized investments. Therefore, if the cost of pledging full collateral is sufficiently low, the market may revert to simple debt contracts even when equity-like contracts are more efficient. In particular, full collateral contracts are likely to undermine the role of intermediaries in markets for less risky assets, and intermediaries are likely to continue trading the riskiest assets without collateral.

Finally, we show that central credit information agencies (a generalized version of credit rating agencies) may relax the requirement of local monopolism, but do not eliminate the need for intermediaries.

1.1. **Related literature.** Recent papers on social and economic networks analyze the efficiency of intermediated trade in exogenously determined networks. Gofman (2011) models over-the-counter markets as trading networks and finds that trading by strategic agents can result in an inefficient equilibrium allocation. Similarly, Condorelli and Galeotti (2011) consider a model in which agents engage in bilateral trading for a single object, and find that adding links to the network may be detrimental to total welfare. In the context of commodities markets, Nava (2010) studies quantity competition in economies in which the set of feasible trades can be described by a network. Nava shows that no economy in which goods are resold can ever be competitive; and that large, well connected economies are competitive. In contrast, our model allows for choices of both repayment (i.e. trade) and relationship choices, and provides insights into the co-evolution of networks and behavior, and in particular into the emergence of financial networks with only a handful of large active intermediaries.

Notably, most of the economic literature on networks makes the assumption that agents have complete knowledge of the network structure, and that they perfectly observe any change made to the network structure throughout the game.[6] The perfect observability approach is often justified as a good approximation for setups in which observability may be imperfect and knowledge of the network structure incomplete. This is not true in our setup: we show that robustness provides a significant refinement relative to the set of networks that can be sustained in a subgame perfect equilibrium of the complete knowledge game.

Recently, several papers take an incomplete knowledge approach: Caballero and Simsek (2010) endow agents (in their case banks) with knowledge of the networks structure up to a permutation on the identities of agents. McBride (2006), Jackson and Yariv (2007), Galeotti

[6]See Goyal (2007) and Jackson (2008) for extensive surveys of the literature on social and economics networks.

et al. (2010), Fainmesser (2011), Fainmesser and Goldberg (2011), and Fainmesser (2012) assume that each agent knows the network structure up to a constant geodesic distance from her in the network. An agent's belief on parts of the networks that are farther away from her may be grounded in a random process that all agents believe to have generated the network (e.g. Jackson and Yariv 2007, Fainmesser 2011, Fainmesser and Goldberg 2011, Fainmesser (2012), and implicitly also Galeotti et al. 2010), or it is allowed to be any belief that corresponds to the information available to the agent (e.g. McBride 2006).

Considering agents who know the network structure up to a fixed geodesic distance provides a mathematically appealing setup and is a novel approximation for social networks and other markets that consist of ex-ante homogeneous agents. However, it is not always obvious what is the observable geodesic distance in a network. Moreover, in markets in which agents are ex-ante different and activity is asymmetric, there are good reasons to expect that agents' knowledge of the network structure depend on their roles in different interactions.

Endogenizing agents' knowledge of the network structure in which they are embedded is an open challenge. Given any realistic dynamic interaction, characterizing the mapping from agents' observations in their own bilateral interactions to their knowledge of the network structure is an intractable exercise. To overcome the hurdle, we offers a dynamic setup in which the following question is much more tractable: what can agents learn about the network structure given what they would observe in their financial interactions over an infinitely long period of time? The answer provides an upper bound on the knowledge that agents may have at any point in time.[7] We derive this upper bound on agents' knowledge in our setup and endow agents with the corresponding knowledge at any point in time. We then characterize networks that are robust in the sense that they can be sustained indefinitely in a pure strategy perfect Bayesian equilibrium of the game given the aforementioned knowledge and (almost) any belief profile.

The robustness requirement allows us to provide sharp predictions in a repeated games setup by ruling out financial networks that are 'too sensitive' to the underlying agents' beliefs. This ties back to a well known challenge in the study of games of incomplete information – separating out equilibria that rely on 'unreasonable' or 'unrealistic' beliefs.[8] Instead of approaching directly the problem of defining reasonable versus unreasonable beliefs in our setup, we suggest a criterion that rules out any network that can be sustained only given highly specific belief profiles. In this sense, this paper offers predictions based on an incomplete information refinement of the standard repeated games model.

Finally, this paper contributes also to the literature on social capital that studies the ability of a society to foster trust and cooperation among its members. Broadly speaking, the literature

[7]In our setup it is also true that once the network structure stabilizes for a sufficient amount of time, agents are bound to reach arbitrarily close to this level of knowledge.

[8]See also the discussion of equilibrium refinements in chapter 8 of Fudenberg and Tirole (1991), and the literature on robust partial implementation, e.g. Bergemann and Morris (2005).

emphasizes two structural elements that generate social capital: on the one hand, the importance of social pressures for fostering cooperation dates back to sociological work by Simmel (1950) and Coleman (1988). This literature emphasizes the importance of closure – in order to facilitate a "strong tie" between two agents, they are required to share many acquaintances. This literature has recently gotten more rigorous theoretical underpinning in the economics literature (see also Raub and Weesie 1990, Haag and Lagunoff 2006, Ali and Miller (2012a), Mihm, Toth, and Lang 2009, Jackson, Rodriguez-Barraquer, and Tan 2011, and Lippert and Spagnolo 2011). On the other hand, the seminal work of Burt (1992) suggests that social capital requires intermediaries that bridge across communities and facilitate the interaction across otherwise disconnected individuals. However, the reason for the emergence of such 'structural holes' that can be exploited by well positioned intermediaries to extract surplus remains mostly unexplored in the economics literature. We analyze an economy in which market participants enforce cooperation informally, and suggest that they must rely on well positioned intermediaries, or intermediators. To that extent our analysis contributes to the understanding of the notion of social capital by showing why individuals may have to limit their direct relationships and rely on highly connected intermediaries to execute transactions on their behalf.

2. A NETWORK-BASED MODEL OF FINANCIAL INTERMEDIATION

Consider a market with a finite set of agents, denoted by V. Each agent is either a lender, an intermediary, or a borrower. The corresponding sets are denoted L, I, and B, and members of the sets are denoted ℓ, i, and b respectively. Time is continuous and agents have a common discount rate ρ. Borrowers come up with investment opportunities stochastically over time. An investment opportunity requires one unit of liquidity and has stochastic returns. Since our main interest is the incentives of borrowers to repay lenders for their investments (rather than borrowers' ability to repay), we abstract from the investment screening problem and assume that all investment opportunities are identical and worth taking: with probability q the return is $\frac{1+r}{q}$ (for some $r > 0$) and with probability $1-q$ it is zero. For the same reason, and following much of the finance literature on intermediation (e.g. Diamond 1984, Babus 2010) we focus on the case that a borrower with an investment opportunity must transact with a lender who has unit liquidity at the same time in order to exploit the opportunity, i.e. borrowers do not have liquid assets, and investment opportunities are time sensitive and are not transferable over time (in section 6.2.1 we extend our results to the case that borrowers can self-finance investments at a cost).

The events that lenders have unit liquidity also arrive stochastically over time. Thus, the event of interest is when a borrower has an investment opportunity and at the same time a lender has unit liquidity. We capture the joint arrival of liquidity and investment opportunities as follows: for any lender ℓ and borrower b, the event that b has a risky investment opportunity and ℓ has one unit of liquidity occurs according to a Poisson arrival process with parameter λ.[9] Intermediaries are agents who may obtain funds from lenders and invest them with borrowers.

Agents are connected by a financial network described by a graph $G =< V, E >$ where $E \subseteq (L \times B) \cup (L \times I) \cup (I \times B)$ is the set of connections (or links/edges). A link always connects two agent of different types (lender-borrower, lender-intermediary, or intermediary-borrower). The network is determined by exogenous factors as well as agents' decisions, and evolves over time as described below. Informally, if at a given time t two agents are connected, it means that they are *able and willing* to transact with each other if the opportunity rises at time t. To highlight the result that some links may not exist because certain lenders (intermediaries) expect that a certain borrower will strategically default on an investment, we assume throughout that there is no cost associated with maintaining a link. It will become clear that this implies that a link is eliminated only if a lender or intermediary at the end of the link expects a default that can be prevented by eliminating the link (our results also hold if maintaining links is costly). We now make the notion of a connection more formal.

Denote by $L_i^G = \{\ell \in L | \ell i \in G\}$ the set of lenders connected to intermediary i in network G, and define similarly L_b^G, I_b^G, I_ℓ^G, B_i^G and B_ℓ^G. We drop the superscript G when it is clear from the context. We say that ℓb^G is true if and only if ℓ and b are connected in G, and that $\ell i b^G$ is true if and only if both ℓ and b are connected to i in G ($\ell i \in G \, \text{and} \, i b \in G$). Similarly, $\ell I b^G$ is true if and only if ℓ and b are connected or there exists at least one intermediary i such that both ℓ and b are connected to i ($\exists_{i \in I} [\ell i \in G \, \text{and} \, i b \in G]$). If $\ell I b^G$ we say that there is an *investment path* between ℓ and b and that they are *financially related*. We denote by $S_b^G = \left\{\ell' | \ell' I b^G\right\}$ the *financial support* of borrower b, i.e. the set of lenders who are financially related to b.

A lender ℓ with unit liquidity who is connected directly to a borrower b (who has an investment opportunity) is able to invest money directly with b with no transaction cost.[10] An intermediary who is connected to a lender ℓ and a borrower b is able to execute an investment on behalf of ℓ by investing ℓ's money with b. To emphasize the role of intermediaries in preventing strategic default, we assume that an intermediary incurs a cost of $c \geq 0$ upon executing an investment. Given that direct investment is costless, this implies that our results hold even if intermediaries have cost disadvantages in executing investments.

[9] Poisson arrival rate implies asynchronicity: at any given moment in time, the probability that more than one borrower has investment opportunity and more than one lender has liquidity is zero. This allows us to focus on intertemporal investment relationships and abstract from other intratemporal consideration that may be second order to sustaining long term relationships. In a more general model with simultaneous arrivals, transactions may still be separable if some investments opportunities are not appealing to certain liquidity holders at certain times, perhaps due to portfolio considerations.

[10] Adding positive transaction costs for lenders and borrowers does not change our analysis.

We follow much of the literature on contract theory and on financial intermediation and assume limited liability and limited availability of collaterals.[11] Therefore, to satisfy liquidity constraints repayment must be at least partially conditioned on investment outcome.[12]

To focus our analysis on the incentives of borrowers to repay, rather than add unnecessary complications via the pricing mechanism, we take the structure of lending contracts as given in the first part of the paper, and focus on *fixed-rate equity contracts* that take the following form: suppose that at time t lender ℓ invests one unit of liquidity with borrower b, then:

(1) If ℓ invests directly with b, and if the investment opportunity has a positive outcome $\left(\frac{1+r}{q}\right)$, then b is required to pay $\frac{1+\phi r}{q}$ to ℓ.
(2) If ℓ invests via an intermediary i, and if the investment opportunity has a positive outcome, then b is required to pay $\frac{1+\phi r}{q}$ to i who upon receiving the payment is required to pay $\frac{1+\phi r-r_I}{q}$ to ℓ, and keep to herself an intermediation fee of $\frac{r_I}{q}$.[13]
(3) If the investment opportunity has a negative outcome, then b is not required to pay to ℓ or i.

That is, prices are fixed over time and across investment paths, e.g. by regulation. In section 2.1 we offer further discussion of the assumption, and in section 8 we extend the model to allow for price competition and bargaining.

Outcomes of investment opportunities are not verifiable. Therefore, the aforementioned contracts are not enforceable by court, and (as in Hart and Moore 1998 and Bolton and Scharfstein 1990) there is room for *strategic default* – e.g. a manager may divert funds from the investment to herself, preventing the firm from repaying a loan. Money transfers are verifiable and enforceable by court. Thus, intermediaries cannot strategically default. We assume further that the money transfers and the outcome of an investment opportunity are *costlessly* observable to the borrower, lender, and intermediary involved in the transaction (but not to any other agent).[14] Therefore, there is no exogenous cost advantage for intermediaries in monitoring investments' outcomes.

2.1. **The game.** In this section we introduce the structure of the game, and offer a discussion of key modeling decisions. To verify that our results are correctly attributed to the (lack of)

[11]The important assumption is that full collateral is costly. In section 6 we generalize our model to allow for self-financing and for collateralized debt. Our results remain qualitatively intact.
[12]Debt contracts are also possible. However, in this case there is a difference between a liquidity default that occurs when a borrower does not have the funds to repay a loan, and a strategic default that occurs when a borrower refuses to pay a loan despite having the funds (see also Bolton and Scharfstein 1990). We focus on the ability to prevent strategic default via repeated interactions.
[13]Liquidity constraints imply that $\phi \in \left[-\frac{1}{r}, 1\right]$, and $r_I \in \left[0, 1+\phi r\right]$.
[14]It is often assumed in the finance literature that strategic default is observable to the manager and the investors. However, it is acknowledged that default may not be observable to outside parties. Thus, it may be impossible to prove that strategic default took place. On the other hand, money transfer across firms leave a paper / electronic communication trail. See also Bolton and Scharfstein (1996), and Babus (2010) for similar assumptions.

observability of financial connections, we present a framework that is flexible enough to accommodate different assumptions with respect to the observability of the network structure.

Notation: the observability operator. Consider an agent j and let K_j map any network G to a set of networks $K_j(G)$ that agent j can consider possible given her observations if the underlying network is G. For example, if for every network G, $K_j(G) = \{G\}$ then we say that agent j perfectly observes the network structure.

At time $t = 0$ the game begins with an initial network $G^0 = \langle V, E^0 \rangle$ being generated by an arbitrary process $\mathscr{G}$ that is unknown to the agents.[15] Note that the set of agents $V = \langle L, I, B \rangle$ is also generated at time $t = 0$. Then, agents simultaneously adjust their links in the following way: each lender ℓ observes $K_\ell(G^0)$ and chooses which of his (observed) links to maintain and which to eliminate, and each intermediary i observes $K_i(G^0)$ and chooses which of her (observed) links to borrowers to maintain and which to eliminate.[16]

Subsequently, any time $t > 0$ begins with a network G^t in place and then the following happens (instantaneously in order):

(1) If lender ℓ has unit liquidity and borrower b has an investment opportunity, then
   (a) Borrower b posts a public liquidity request observable to all of the agents in the market. Simultaneously, lender ℓ searches for investments opportunities with all of the borrowers and intermediaries connected to him (I_ℓ and B_ℓ).[17]
   (b) Any intermediary i that is connected to both ℓ and b informs both that she is able to intermediate the transaction (e.g. ℓ and b can view each other's posting on the intermediary's platform, i can contact them with a reply to their search, etc.).
   (c) Investment goes through using the contract that maximizes the combined expected payoffs of ℓ and b. If more than one such contract exists, then one such contract is chosen uniformly at random (under the assumption of uniform fees this implies that if ℓ and b are connected to each other they transact directly with each

[15] Allowing agents to know the process $\mathscr{G}$ does not change our results, but makes the analysis more notationally intensive. See also section 2.3.1.

[16] An alternative is to assume that there is no initial underlying network and that at $t = 0$ agents simultaneously establish links. This can be done by adopting the notion of a conjectural equilibrium. See Battigalli, Gilli, and Molinari (1992), and Gilli (1999) for more details on conjectural equilibria, and McBride (2006) for an application of conjectural equilibria to the study of network formation.

[17] In the imperfect observability setup below, at time $t = 0$ (before any investment takes place), it is not obvious that borrower b knows which intermediaries and lenders are willing to invest in his investment opportunities. Therefore, it is reasonable to assume that a borrower tries to finance publicly until he realizes who are his potential investors. For simplicity, we maintain the public financing attempt at any time t, but our results extend immediately if each borrower tries to finance publicly at least once and finances either publicly or via connections beyond that one time. On the other hand, even in the imperfect observability setup, at any time t, lender ℓ knows what channels of investments he is willing to take and can restrict his search for investments to these channels at any $t > 0$.

other, and otherwise they choose uniformly at random between available intermediaries).[18]

(d) If investment occurs and has a positive outcome $\left(\frac{1+r}{q}\right)$ then b decides whether to make payments according to the contract.

(e) If b does not make payments due directly to ℓ (via i) the link between b and ℓ (i) is eliminated permanently from the network and the new network is denoted G^{t+}.

(f) Time t consumption occurs – no liquidity is transferred to any time $t' > t$.

(2) Network adjustments:

(a) Any lender ℓ who observes a change in the network structure at time t (i.e. $K_\ell\left(G^{t+}\right) \neq K_\ell\left(G^{t}\right)$) chooses which of his links he maintains and which he eliminates. Similarly, any intermediary i who observes a change in the network structure at time t (i.e. $K_i\left(G^{t+}\right) \neq K_i\left(G^{t}\right)$) chooses which of her links to borrowers she maintains and which she eliminates. Denote the new network by G^{t++}.

(b) If any lender or intermediary eliminates a link at stage 2(a) $\left(G^{t++} \neq G^{t+}\right)$ then lenders and intermediaries that observe the changes can adjust their network structure. Formally, let $G^t = G^{t+}$ and $G^{t+} = G^{t++}$ and repeat step 2.

We are interested in characterizing the set of initial networks that can be sustained indefinitely with probability 1. To this end, the game focuses attention to one type of choices made by each type of agents: lenders choose who they trust with their money, intermediaries choose who they trust with the lenders' money (and their own fees), and borrowers choose whether to make payments according to the agreed contract. Because there is no exogenous cost of sustaining links, it will become clear that lenders and intermediaries eliminate links only if they expect that eliminating the links will prevent strategic default from happening.

Several restrictions are imposed by the structure of the game. First, once a borrower defaults on a lender (intermediary) the lender (intermediary) ceases to invest with the borrower and the relationship cannot be revived. This simplifies the analysis in that it eliminates complicated forms of punishment where an agent who observes a default disconnects a link just in order to communicate the defection and trigger further punishments by others while he himself returns to invest with the defaulting borrower almost instantaneously. This assumption can be motivated using behavioral reasoning, or by considerations of organizational economics (e.g. the accountability of a manager in an organization for investing with a borrower that previously defaulted may be different than for investing with a borrower with a clean record).

Second, we do not consider the formation of new links, but only the dissolution of links. This embodies the idea that the formation of new relationships is a longer-term process (due diligence, establishing a personal trust relationship, etc.), and that the decision to make payments

[18]Assuming that a lender and a borrower choose among payoff equivalent investment paths uniformly at random is done for analytical convenience only. For our results to go through it is sufficient to assume that there is some positive probability that any of the payoff equivalent paths is chosen.

and/or punish an agent can be taken more quickly.[19] It is important to note that we do cover the case where the market starts with the initial network G^0 being the complete network as well as any other network, so we do not a priori restrict the links that might be formed, and so our results do make predictions about which financial networks can be sustained in a market.[20] The important restriction is that an agent who has lost a relationship cannot (quickly) replace it with a newly formed one.

Moreover, with the exception of time $t = 0$, agents are assumed to destroy links only when they are exposed to information about a default or a structural change in some part of the network. This regularity requirement embodies the idea that people do not break (trust based) relationships arbitrarily when their positive expectations are met, but rather break relationships in response to a default or to a change that they observe in the environment (e.g. change in the network structure). In that sense, maintaining a link is a promise to maintain the link as long as all goes well in the entire market, and can be thought of as decision to extend a *line of credit* - a tentative commitment to deliver funds when necessary and possible.[21] This assumption can also be motivated using considerations of organizational economics or even procedural/legal considerations, e.g. a worker may need to provide good reasons to convince her manager to make a change in a firm's credit and investment policies. Notably, this does not rule out ostracism or the spread of punishment via contagion – after any change in the network structure, agents have time for any sequence of network adjustments before any opportunity to transact comes up. However, it is a substantial assumption: it rules out "bad" equilibria in which a borrower b defaults in a transaction with a lender or intermediary, say j, based on an expectation that the relationship with j will be eliminated independently of j's belief with respect to the network structure and of any financial transaction between any lender, intermediary, and borrower in the market.

Finally, we impose a restriction on prices. The restriction can be motivated by legal regulation or social norms, yet its theoretical implications deserve further discussion. In a repeated games setup, any bargaining game inevitably allows for the existence of equilibria in which ϕ, and r_I are complex objects that depend on ℓ, i, b, $\left\{G^{t'}\right\}_{t'\leq t}$ and even directly on t in arbitrary ways. While theoretically intriguing, there are many reasons (behavioral, organizational, and legal) that prevent such complicated intertemporal bargaining from being the norm in financial markets. Potentially for that reason, such considerations are absent from the related finance literature. We hope that by focusing on specific equilibrium prices, we are able to make

[19]see Jackson, Rodriguez-Barraquer, and Tan (2011) for a similar assumption in the context of favor exchange.

[20]In the complete network all of the lenders are connected to all of the intermediaries and borrowers in the economy, and all of the intermediaries are connected to all of the borrowers in the economy.

[21]Lines of credit are often extended by banks, financial institutions and other licensed consumer lenders to creditworthy customers. A line of credit is effectively a bank account that can readily be tapped at the borrower's discretion. Lines of credit can be secured by collateral or unsecured, and often have clauses that allow lenders to cancel the lines following big changes in the market environment.

the implications of our informational assumptions more transparent in the context of the related finance literature. In section 8, we extend the set of equilibria considered and allow for meaningful bargaining and price competition.

All of the aforementioned restrictions can be relaxed with no change to the results in the benchmark case when financial connections are publicly observable. In the limited observability analysis, the simplifications provide tractability in a problem that is already very complex and allow us to establish a rich and economically meaningful set of testable predictions.

2.2. **Benchmark: publicly observable financial connections.** The economic literature on networks predominantly make the following two related assumptions: [1] agents have complete knowledge of the network that they are embedded in; and [2] agents observe immediately any change in the network.[22] To motivate our analysis, we first consider these assumptions as a natural benchmark.

Suppose that a borrower b is financially related to only one lender, say ℓ. Then, as long as b and ℓ are financially related, the rate of arrival of investment opportunities for which b manages to raise funds is λ. Therefore, if b intends to always repay according to the aforementioned contract, and expects to remain financially related to ℓ forever, then the present value of b's expected future payoffs is $\frac{1}{\rho}\cdot\lambda\cdot(1-\phi)\,r$. Now recall that S_b^G is the financial support of borrower b (i.e. the set of lenders who are financially related to b), and denote by $|A|$ the number of elements in a set A. Then the expected payoff of a borrower b in a network G that he expects to exist forever is

$$\frac{1}{\rho}\cdot\left|S_b^G\right|\cdot\lambda\cdot(1-\phi)\,r. \tag{2.1}$$

On the other hand, if b defaults then he increases his immediate payoff by $\frac{1+\phi r}{q}$, but may lose some of her connections. Therefore, the following captures the minimal number of lenders whose threat of ostracizing the borrower may be sufficient to make it profitable for the borrower to repay.

$$m = m(\rho,\lambda,\phi,r,q) = min\left\{n\in\mathbb{N}|\,\frac{1}{\rho}\cdot n\cdot\lambda\cdot(1-\phi)\,r \geq \frac{1+\phi r}{q}\right\}. \tag{2.2}$$

The following result follows immediately.

Proposition 1. *There exists a subgame perfect equilibrium such that starting with $G^0 = G$, the network G is sustained indefinitely with probability 1 if and only if each borrower is financially related (in G) to at least m lenders or to none* $\left(\forall_{b|\ell I b^G}\left|S_b^G\right|\geq m\right)$.

[22]Notably, this instantaneous complete knowledge setup implies also common knowledge of the network structure. To abstract from the question of higher order knowledge we maintain throughout the paper that the mapping from any network structure to any agent's knowledge and beliefs with respect to the network structure are common knowledge.

The proof relies on simple bang-bang strategies – if any borrower ever defaults, at least one link is eliminated immediately. Since the elimination of a link is immediately observed by everyone, all lenders and borrowers can coordinate on disconnecting all of their links simultaneously and the entire market shuts down.[23] So for instance, if there are more than m lenders in the economy, the complete network could be sustained. In the complete network every lender is connected to all of the borrowers in the economy, and all investments are executed directly – with no use of costly intermediation.

2.2.1. *Caveats.* The assumption of perfectly observable financial connections is problematic. Even if agents observe the physical restrictions imposed on the network as captured by G^0, it is unlikely that an agent j observes immediately when some lender ℓ is no longer willing to invest with some borrower b if both ℓ and b are in a remote part of the network relative to j. The use of bang-bang strategies only serves to highlight further the unrealistic nature of perfect and immediate observability in this market.[24]

For example, consider Figure 2.1 and assume that the parameters of the model $(\rho, \lambda, \phi, r, q)$ are such that $m = 4$. Now suppose that b defaults on an investment made directly by ℓ_6: how can ℓ_6 be certain that i_1 (or ℓ_5) will punish b? How would ℓ_1 know the exact time that he is supposed to punish? Note that if $\ell_1 - \ell_5$ cannot learn about the default and ℓ_6's punishment, then i_1 prefers to continue to intermediate investments in b. Assuming instantaneous complete knowledge of the network structure as well as relying on bang-bang strategies provides a theoretical explanation, but one that relies heavily on potentially unreasonable assumptions.

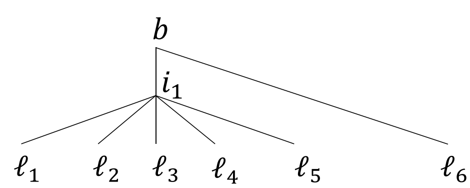

FIGURE 2.1.

2.3. **Unobservable financial connections.** A good alternative to the assumption of publicly observable financial connections is hard to come by. In the language of networks, this amounts

[23]If the parameters of the model $(\rho, \lambda, \phi, r, q)$ are such that $m = 1$ then slightly more subtle enforcement strategies are used in the proof: a default by any borrower is followed by the eliminations of all links between lenders and intermediaries as well as between intermediaries and borrowers (in addition to the link on which the default occurred), but other direct links between lenders and borrowers are not eliminated.

[24]In some markets assuming complete information or almost complete information is realistic, e.g. in the context of favor exchange in small rural villages, Jackson, Rodriguez-Barraquer, and Tan (2011) assume that all agents publicly announce at the beginning of every period who they are willing to provide favors to. This captures the idea that information diffusion in a small village is fast relative to the frequency of transactions. However, even in their setup, Jackson, Rodriguez-Barraquer, and Tan (2011) agree that enforcement using market-wide bang-bang strategies does not provide the correct criteria for predictive purposes.

to assuming that agents have incomplete knowledge of the network that they are embedded in. To-date, the few papers that explore games with incomplete knowledge of the network structure make assumptions directly on what agents know about the network, rather than make a connection between interactions and the acquisition of knowledge.[25] This approach provides a reasonable starting point for the analysis. However, a more satisfactory (and challenging) approach requires an analysis of what agents *can learn* based on their interactions and the information available to them in the market. For example, if over a long period of time an intermediary i intermediates very few investments between a lender ℓ and a borrower b who are connected to her, then i might infer that with high probability, there are additional intermediaries connecting ℓ and b. As long as the network stays fixed, this inference becomes more accurate the longer the period of time used for the inference. In fact, if the time frame considered is very long, then i may have a pretty accurate idea of the number of other intermediaries connecting ℓ and b.

We next define a notion of Perfect Bayesian Equilibrium (PBE) that fits an environment in which agents observe the network structure imperfectly according to a given observability function. Then, we analyze the patterns of interactions described in section 2.1 and propose an *upper bound* on agents' knowledge of the network structure based on their observations. Our goal is to analyze the Bayesian game that is induced by endowing agents with this upper bound as their knowledge, and answer the following question: which financial networks can be sustained indefinitely given any belief profile that is consistent with agents' knowledge?

2.3.1. *Perfect Bayesian Equilibrium with Observability Operators* . Let Ω be the set of all possible states of the world. Each state $\omega \in \Omega$ specifies a set of agents V, an initial network G^0, a time t, and the entire history of play up to time t. Incorporating the time index and history into the state is done for notational convenience only – when it plays a role in the analysis we add a superscript t $\left(\omega^t\right)$.

Denote by $\Omega(G)$ the set of states of the world that are consistent with the underlying network being G and denote by $\omega(G)$ a member of $\Omega(G)$. We also denote by $G(\omega)$ the underlying network in state ω. An information set of agent j is denoted $h_j(\omega)$ (or h_j when ω is clear from the context). A state ω' belongs to $h_j(\omega)$ if j cannot distinguish between ω and ω' (and then $h_j(\omega) = h_j\left(\omega'\right)$). We denote by $H_j = \left\{h_j(\omega)\right\}_{\omega\in\Omega}$ the knowledge partition of agent j. A belief of agent j is a mapping $\mu_j : H_j \to \Delta(\Omega)$ (also written as $\mu_j : \Omega \to \Delta(\Omega)$) where $\Delta(\Omega)$ is the set of probability distributions over Ω. We denote by $\mu = \left\{\mu_j\right\}_{j\in V}$ a belief profile, and by $\mu_j^t = \mu_j\left(\omega^t\right)$.

We now define Perfect Bayesian Equilibrium in an environment in which agents observe the network structure imperfectly according to a given observability function. For a borrower b, let

[25] E.g. McBride (2006), Jackson and Yariv (2007), Galeotti et al. (2010), Fainmesser (2011), Fainmesser and Goldberg (2011), and Fainmesser (2012) assume that each agent knows the network structure up to a constant geodesic distance from her in the network.

$LI_b^G = \{\ell i | \ell i b^G\}$ be the set of lender-intermediary pairs such that ℓ is connected to i and i is connected to b in network G. For a set A, denote by $\mathscr{P}(A)$ the set of all subsets of A. Then, a pure strategy for a borrower b is a mapping $\sigma_b : H_b \to \mathscr{P}\left(LI_b^G \cup L_b^G\right)$ from b's information set to the lenders and intermediary-lender pairs to whom b will make a repayment (when required by the contract) – i.e. if $\ell \in \sigma_b(h_b)$ ($\ell i \in \sigma_b(h_b)$) then borrower b will make the required repayment directly to ℓ (via i) if b's information set is h_b. Similarly, a pure strategy for an intermediary i is a mapping $\sigma_i : H_i \to \mathscr{P}\left(B_i^G\right)$ from i's information set to the set of borrowers to whom i maintains a connection. Finally, a pure strategy for a lender ℓ is a mapping $\sigma_\ell : H_\ell \to \mathscr{P}\left(I_\ell^G \cup B_\ell^G\right)$ from ℓ's information set to the set of intermediaries and borrowers to whom ℓ maintains a connection. We denote by $\sigma = \{\sigma_j\}_{j \in V}$ a pure strategy profile.

We abstract from considerations of higher order beliefs and focus entirely on the case that agents have common knowledge of the observability partitions of all agents $\left(\{K_j\}_{j\in V}\right)$.[26] Therefore, we can define $u_j(\omega)$ to be the expected flow payoff of agent j when the state is ω, and let $V_j(\omega) = E_{\omega^\tau | \omega^t = \omega, \sigma}\left[\int_t^\infty e^{-\rho(\tau - t)} u_j(\omega^\tau)\, d\tau\right]$ be the expected discounted present value of all future payoffs of agent j starting with ω being the state, and given the strategy profile σ.

Definition 1. An assessment (σ, μ) is a Perfect Bayesian Equilibrium with Observability Operators $\{K_j\}_{j\in V}$ (PBE-K) if:

(1) $\forall_{j\in V, \omega\in\Omega, \hat\sigma_j \neq \sigma_j} \left(E_{\omega' | \mu_j, \omega}\left[V_j\left(\omega' | \sigma_j, \sigma_{-j}\right)\right] \geq E_{\omega' | \mu_j, \omega}\left[V_j\left(\omega' | \hat\sigma_j, \sigma_{-j}\right)\right]\right)$;

(2) $\forall_{j\in V, \omega\in\Omega}\ \mu_j(\omega)$ is consistent with Bayes law whenever possible; and

(3) $\forall_{j\in V, \omega\in\Omega}\ \mu_j(\omega)$ puts positive probability only on states that are consistent with $K_j(G(\omega))$, i.e. only on states that belong to $\left\{\omega' | K_j\left(G\left(\omega'\right)\right) = K_j(G(\omega))\right\}$.

The first two conditions are standard in the literature. Condition (3) captures the idea that agent j knows $K_j(G(\omega))$, i.e. even a zero probability event cannot convince j that the state is such that the information in $K_j(G(\omega))$ is wrong.

If for any agent j and network G we have that $K_j(G) = \{G\}$ then we are back in the complete information setup and Definition 1 coincides with the definitions of a PBE as well as of a subgame perfect equilibrium. Otherwise, Definition 1 puts only minimal restrictions on agents' beliefs. An agent may believe that there are different numbers (and identities) of lenders and intermediaries in the market, as well as different patterns of financial connections (i.e. network structure). In particular, Definition 1 does not require that agents know the distribution over initial sets of agents and networks implied by the network generating process $\mathscr{G}$. Consequently, agents may have common or heterogeneous priors. Notably, our results do not rely on whether

[26] With some additional notation, this can be done formally by conditioning agents' payoffs on $\left(\{H_j\}_{j\in V}\right)$, as well as conditioning agents' expectations on $\left(\{K_j\}_{j\in V}\right)$ in part 1 of Definition 1.

we allow for heterogeneous priors or not, and the modeling choice is done for convenience only.[27]

2.3.2. *Inferring the network structure from financial interactions - steady state knowledge.* Consider a network G and assume that it is sustained for a long period of time. Then, each agent can use the frequencies of her own financial interactions with different other agents to learn something about the network structure. Learning may depend on an agent's prior and on the realizations of the stochastic elements in the market (e.g. arrival of investment opportunities). As a result, analyzing agents' learning continuously may be intractable. However, it turns out that the limit of such learning is easier to analyze. That is, as long as an agent starts with a rich enough prior and learns about the network structure only based on her observations in the game, there is some information that over an asymptotically long period of time the agent receives with probability 1. Similarly, there is some information that is never revealed to the agent.

We now explain our notion of steady state observability informally and suggest that it provide a reasonable upper bound on the knowledge of the network that agents are likely to obtain by learning from their observations throughout the game. Consider the following hypothetical exercise: assume that beyond their immediate connections, agents learn about the network structure only from their own financial interactions.

For an agent $j \in V$, let $\mu_j \in \mathcal{M}_j$ if belief μ_j is consistent with Bayes rule, puts strictly positive probability on the true state at time t, and puts probability 1 on the network not changing from time t onwards. Suppose that starting at time t the network does not change and let $K_j^{\infty}\left(G^t\right)$ be a set of networks (including sets of agents V) such that for any network $G' \in K_j^{\infty}\left(G^t\right)$ there exists a belief $\mu_j \in \mathcal{M}_j$ that assigns G' a strictly positive probability after an asymptotically long period of play. We find that all of the networks in the set $K_j^{\infty}(G)$ have several common characteristics that capture j's steady state knowledge of the network and call $K_j^{\infty}(G)$ the set of *steady state believable networks* of agent j (see also Definition 5 on page 37). Being able to characterize what j eventually knows suggests an upper bound on the knowledge that agents can acquire based only on their observations at any moment in time. Any additional knowledge of j must be based on j's beliefs, or on previous knowledge that j has and is not developing endogenously via the interactions studied in our model.

Before turning to the formal analysis, example 1 demonstrates how one can derive as a result what agents can and cannot infer from their observations in a long term interaction in a fixed network.

[27]E.g. a simple setup in which the analysis of the game with a common prior and with heterogeneous priors coincide is when the initial prior is a smooth prior over an infinite set of networks. In this case the requirement that agents beliefs be consistent with Bayes law have no bite at $t = 0$. More generally, our results can be replicated in a common prior framework as long as we allow for sufficient richness on the common prior.

Example 1. *Consider the financial network G in figures 2.2, 2.3, and 2.4, and assume that G is sustained indefinitely. At some point in time, the event that b_1 has an investment opportunity and ℓ_3 has unit liquidity must occur. At that time, borrower b_1 is approached by lender ℓ_3 directly, and also i_1 offers to intermediate the transaction between ℓ_3 and b_1. From this event, and the permanent absence of other trade opportunities, b_1 eventually learns (with arbitrarily high probability) that he is connected only to i_1 and ℓ_3, and that the only lender that i_1 is connected to is ℓ_3. In addition, because all borrowers declare publicly when they have investment opportunities, b_1 (like any other agent in the network) gets to know the set of borrowers B.*

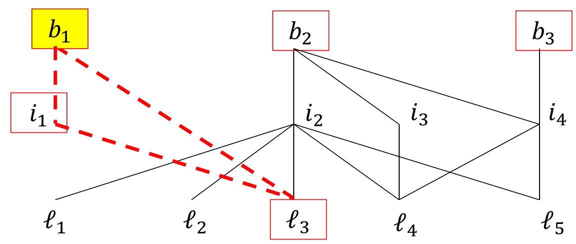

FIGURE 2.2. The knowledge of the network – borrower b_1. Suppose that the network in the figure is sustained indefinitely. Rectangles mark the agents whose identities must eventually be revealed to borrower b_1, and dashed lines represent the links that borrower b_1 must eventually know to exist.

Similar reasoning can be applied to derive that lender ℓ_4 eventually learns that [1] he is connected only to i_2, i_3 and i_4; [2] the only borrower that i_2 and i_3 are connected to is b_2; and [3] the only two borrowers that i_4 is connected to are b_2 and b_3.

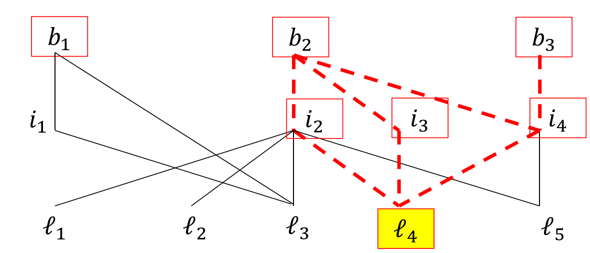

FIGURE 2.3. The knowledge of the network – lender ℓ_4. Suppose that the network in the figure is sustained indefinitely. Rectangles mark the agents whose identities must eventually be revealed to lender ℓ_4, and dashed lines represent the links that lender ℓ_4 must eventually know to exist.

Interestingly, we find that intermediaries must eventually learn about the existence of some investment paths that do not pass through them. For example, the event that b_2 has an investment opportunity and ℓ_4 has unit liquidity occurs an indefinite number of times. Each time, the probability that i_4 gets to intermediate the transaction is $\frac{1}{3}$. Thus, i_4 eventually learns that there are three distinct investment paths between b_2 and ℓ_4, and that the three paths involve three distinct intermediaries (one of whom is i_4 herself). Similarly, i_1 is able to learn that b_1 and ℓ_3 are directly connected to each other.

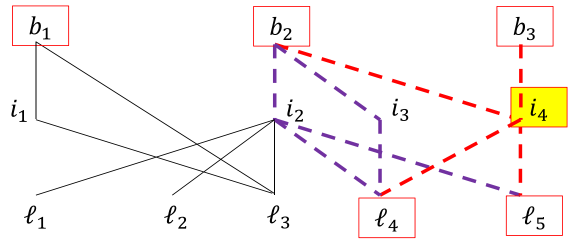

FIGURE 2.4. The knowledge of the network – intermediary i_4. Suppose that the network in the figure is sustained indefinitely. Rectangles mark the agents whose identities must eventually be revealed to intermediary i_4, and dashed lines represent the links that intermediary i_4 must eventually know to exist. Interestingly, i_4 can learn that other investment paths connecting b_2 and ℓ_4 (or ℓ_5) exist, but cannot learn the identity of the intermediaries along those paths.

The formal result requires additional notation and definitions that are not used in the remainder of the paper and are deferred to the Appendix together with the result itself (see Definition 5 and Proposition 5). Notably, the intuition from example 1 generalizes and the following is true for any believable network $G' = \langle V', E' \rangle \in K_j^{\infty}(G)$: V' has the same set of borrowers as V; if j is a lender then the set of intermediaries and borrowers connected to j, as well as the set of borrowers connected to any intermediary who is connected to j are the same in G' and G; if j is a borrower then the set of intermediaries and lenders connected to j, as well as the set of lenders connected to any intermediary who is connected to j are the same in G' and G; and finally, if j is an intermediary then the set of lenders and borrowers connected to j are the same in G' and G, and additionally, for each ℓ and b who are connected to j, ℓ and b are connected directly to each other in G if and only if they are connected directly to each other in G', and if ℓ and b are not directly connected to each other in G, then the number of distinct intermediaries who connect ℓ and b is the same in G and G'.[28]

We denote by PBE-K^{∞} a PBE-K such that $\{K_j\}_{j\in V} = \left\{K_j^{\infty}\right\}_{j\in V}$, and focus on pure strategy PBE-K^{∞} as our solution concept for the remainder of the paper. A pure strategy PBE-K^{∞} always exists – the proof follows directly from Lemma 1 below. We denote by $\Sigma(G)$ the set of pure strategy PBE-K^{∞} for which starting with $G^0 = G$ the network G is sustained indefinitely with probability 1. A novel aspect of this approach is that it provides an endogenous upper bound on knowledge of the network that agents can obtain by observing their own financial interactions.

[28]Knowledge of the exact number of distinct intermediaries who are connected to ℓ and b is not important. For our results to hold it is be sufficient that an intermediary knows if at least one more intermediary is connected to ℓ and b. Thus, assuming that a lender and a borrower choose among payoff equivalent investment paths uniformly at random, which pins down the specific knowledge structure, is done for analytical convenience only.

3. ROBUST NETWORKS

Requiring that a network be sustained indefinitely in *some* pure strategy PBE (i.e. some strategy profile σ and some belief profile μ) allows for the existence of implausible financial relationships that are supported by very specific and somewhat 'unreasonable' belief profiles combined with strategies that depend heavily on those beliefs.[29]

Instead, we now provide a complete characterization of *robust networks* – networks that can be sustained in pure strategy PBE-K^{∞} of the infinitely repeated game given any belief profile from a large set of belief profiles, and relying only on strategies that are not 'too sensitive' to agents' priors. For simplicity, for the remainder of the paper we assume away non-generic parameter values for which Assumption 1 does not hold. All proofs are deferred to the Appendix.

Assumption 1. $\forall_{n\in\mathbb{N}}\frac{1}{\rho}\cdot n\cdot\lambda\cdot\left(1-\phi\right)r\neq\frac{1+\phi r}{q}$.[30]

3.1. A simple special case: one borrower.

We begin by analyzing a special case in which there is only one borrower in the economy. The one borrower case highlights many of the insights while avoiding some of the complexities. The general multi-borrower case is deferred to section 3.2.

A natural candidate for the set of belief profiles to be considered is the set of all belief profiles. However, even with one borrower, we cannot rule out the existence of belief profiles that make it impossible to sustain any network. For example, consider a lender ℓ who is connected to one intermediary i who is in turn connected to borrower b. Lender ℓ observes very little of the network structure and we cannot rule out the possibility of a belief profile that facilitates only PBE-K^{∞} in which ℓ disconnects from i at time 0. Notably, such a belief profile must be very specific and exhibit a particular combination of beliefs of ℓ and i; otherwise ℓ would trust i to decide based on i's knowledge. In addition, the fact that this reasoning can eliminate any network precludes any discrimination between networks that are 'more' and 'less' robust. Thus, we focus on the largest set of belief profiles for which initial trust by lenders is possible, and define robust networks to be networks that can be sustained given any such belief profile.

Definition 2. Belief profile μ is a *Lending Enabling Belief profile* (LEB) for network G if there exists a strategy profile σ such that $\left(\sigma,\mu\right)$ is a pure strategy PBE-K^{∞}, and if $G^0=G$ then according to σ all lenders keep all of their links to intermediaries in G at time 0 $\left(\forall_{\ell}\sigma_{\ell}\left(\omega^0\left(G\right)\right)\supseteq I_{\ell}^{G}\right)$. Denote by $LEB\left(G\right)$ the set of all LEBs for network G. A network G is *LEB-robust* if for any belief profile $\mu\in LEB\left(G\right)$ there exists a pure strategy PBE-K^{∞} such that starting with $G^0=G$, the network G is sustained indefinitely with probability 1 $\left(\forall_{\mu\in LEB(G)}\exists_{\sigma}\left(\sigma,\mu\right)\in\Sigma\left(G\right)\right)$.

[29]The set of networks for which there exist pure strategy PBE-K^{∞} in which they are sustained indefinitely is characterized in section 5.2.2. The interested reader is also referred to Example 4 on page 38 in the Appendix, which demonstrates what we mean by *very specific and somewhat 'unreasonable' belief profile*s.

[30]I.e. there is no number of lenders n such that a borrower is exactly indifferent between her payoff from a single default and her payoff from long term cooperation with exactly n lenders.

By definition, if a belief profile μ is not an LEB of network G, then there is no pure strategy profile σ such that $(\sigma,\mu)\in\Sigma(G)$. In that sense, the set $LEB(G)$ is the largest set that can be considered – there is no set of belief profiles that contains a belief profile $\mu'\notin LEB(G)$ such that network G can be sustained in equilibrium given any belief profile from the set. We also verify that the set $LEB(G)$ is never empty.

Lemma 1. *For any G, there exists an LEB. Moreover, there exists a belief profile μ which is an LEB for all networks* $(\exists_{\mu}\forall_{G}\mu\in LEB(G))$.

We provide a sketch of the proof for the case where the parameters of the model are such that $m>1$. Consider any belief profile such that at time $t=0$: [1] if a lender ℓ is connected to an intermediary i who is connected to the borrower, then according to the belief of ℓ, with probability 1, there are at least m lenders who are connected only to i; and [2] according to the belief of any intermediary i, with probability 1, for any intermediary $i'\neq i$ who is connected to the borrower there are at least m lenders who are connected only to i'. Given any such belief profile the following strategy profile constitutes a pure strategy PBE-K^{∞} in which at time $t=0$ all of the lenders keep all of their links to intermediaries: [1] all lenders always keep all of their links to intermediaries and eliminate their direct connections to b; [2] an intermediary i keeps her link to the borrower if and only if i is connected to at least m lenders whose only investment path to b is via i; and [3] the borrower makes the agreed upon payments to an intermediary i if and only if i is connected to at least m lenders whose only investment path to b is via i.

3.1.1. *Locally monopolistic networks.* It turns out that LEB-robust networks have a particular structure.

Definition 3. An intermediary i (lender ℓ) is a *k-local monopoly* in G if for any investment path ℓib (ℓb) in G there exist at least k lenders whose only investment path to b passes via i (ℓ). A network G is *k-locally monopolistic* if all the lenders and intermediaries in G are k-local monopolies in G.

By definition, a lender that is connected directly to b cannot be a k-local monopoly for any $k>1$, so the network in Figure 2.1 on page 13 is 1-locally monopolistic even though the intermediary in the network is a 5-local monopoly. Figure 3.1 provides additional examples of k-locally monopolistic networks for different values of k.

Theorem 1. *Consider an economy with a single borrower. A network is LEB-robust if and only if it is m-locally monopolistic.*

If the parameters of the model are such that $m>1$, then Theorem 1 implies that there are no direct links between lenders and borrowers in any LEB-robust network. Thus, if $m>1$ all investments must be intermediated. However, Theorem 1 (and the more general Theorem 2 below)

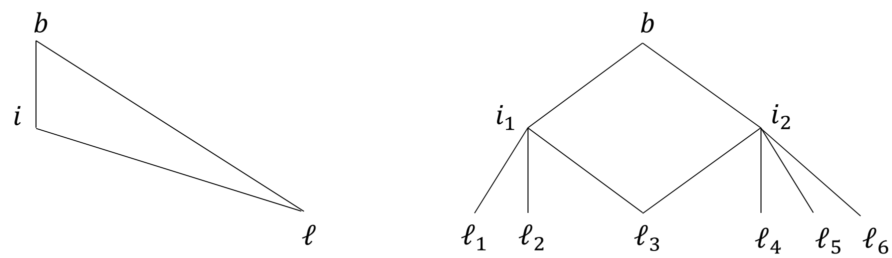

FIGURE 3.1. Locally monopolistic networks. In the leftmost network both i and ℓ are only 0-local monopolies. Therefore, the network as a whole is 0-locally monopolistic. In the rightmost network, i_1 is a 2-local monopoly whereas i_2 is a 3-local monopoly. Thus, the network as a whole is 2-locally monopolistic (but not 3-locally monopolistic).

provides a more complete picture. In order for an intermediary i to successfully incentivize a borrower b to make his repayments, i must provide b with unique access to at least m lenders. In that, our results provide new insights that are related to the discussion of the optimal size of financial intermediaries: in our model, an intermediary is effective (and needed) if she provides each of her borrowers with a sufficient stream of funds that would not have been accessible to the borrower otherwise. Thus, it is not the absolute size of an intermediary, or the overall number of investments that she intermediates. It is rather the exclusivity over a sufficient number of investment paths. Such exclusivity can be achieved by a large intermediary, but it can also be achieved by an intermediary who specializes by focusing on a small (but not too small) number of borrowers and lenders that are not financially related otherwise. For example, an intermediary can focus on local businesses, or can provide a connection between otherwise disconnected communities of lenders and borrowers.

3.1.2. *Sketch of the proof of Theorem 1.* A subtle observation is at the heart of the proof of Theorem 1: if at any time $t > 0$ an intermediary i is an m-local monopoly, then in *all* pure strategy PBE-K^∞, i keeps the link with b indefinitely, and b never defaults on i. This is true because [1] if an intermediary is an m-local monopoly, this is common knowledge to the intermediary and the borrower; and [2] given that links are eliminated only by agents who observe a default or a change in the network, an intermediary can effectively signal to the borrower that she is willing to intermediate investments on the borrower's behalf as long as the borrower does not default on her. The formal claim is available in Lemma 2 in the Appendix.

Returning to Theorem 1, it is now relatively simple to show that m-locally monopolistic networks are LEB-robust. More challenging is to show that *only* m-locally monopolistic networks are LEB-robust. The reason that this is true lies in the knowledge of the network structure that intermediaries (do not) have. This is demonstrated in Example 2.

Example 2. *Consider the two networks in Figure 3.2 and assume that the parameters of the model are such that* $m = 3$. *Note that the two networks are observationally identical from the point* *of view of intermediary i_1, i.e. they induce the same $K^{\infty}_{i_1}(\bullet)$. If all agents know that they are in the leftmost network, there exists a pure strategy PBE-K^{∞} in which the leftmost network is sustained indefinitely – if b defaults on i_1 then i_2 observes the elimination of the link $i_1 b$ and in the remaining network b would have access to liquidity only from two lenders, thus i_2 eliminates the link $i_2 b$ in response.*

However, this is not the case for other belief profiles. Consider a belief profile such that i_1 and all of the lenders ($\ell_1 - \ell_3$, as well as the hypothetical ℓ_4 and ℓ_5) assign probability 1 to the network being the rightmost network (in which i_2 is a 3-local monopoly). If the network is the rightmost network, then if ℓ_3, ℓ_4, and ℓ_5 keep their links at time $t = 0$, and if i_2 keeps her link to b at any time t, then the best strategy of the borrower is to default on i_1. To verify that the leftmost network is not LEB-robust, it is then sufficient to show that in any equilibrium in which the leftmost network is sustained, it is also the case that if the network is the rightmost one then all of the lenders connected to i_2 keep their links at $t = 0$, and and i_2 keeps her link to b at any time t.

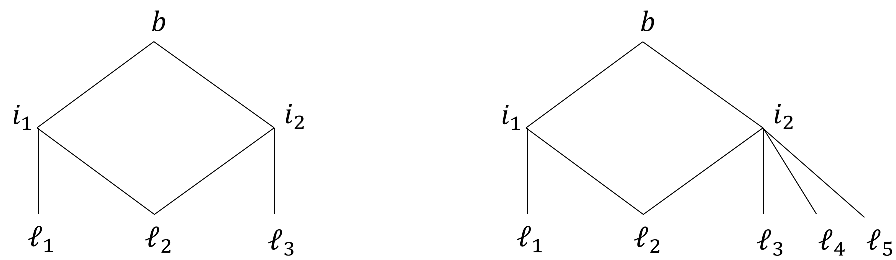

FIGURE 3.2.

3.2. **The general case: many borrowers.** The argument used to show that *only* m-locally monopolistic networks are LEB-robust in the one borrower case extends immediately to economies with any number of borrowers.

Corollary 1. *A network is LEB-robust only if it is m-locally monopolistic.*

On the other hand, when there are many borrowers, the set of LEBs is too large to allow for a straightforward extension of the approach used in section 3.1 to show that all m-locally monopolistic networks are LEB-robust. A closer examination of the set of LEBs reveals the reason that some m-locally monopolistic networks may not be LEB-robust: the strategy space considered in evaluating whether a belief profile is an LEB includes also strategies that are somewhat 'contrived' in a well defined sense. In what follows we explain in more details the nature of the difficulty and then provide a refinement of the set of LEBs that allows us to generalize the insights of Theorem 1. Less theoretically inclined readers may wish to skip directly to Theorem 2 on the next page.

The key difficulty is due to the fact that a borrower b who is connected to an intermediary i does not know how many other borrowers intermediary i is connected to. This lack of common knowledge between b and i prevents any signaling of intentions by i, i.e. there may be an equilibrium in which off the equilibrium path i decides whether to keep her link to b based on actions of other borrowers that i is connected to, even though b does not observe the actions or connections of other borrowers. This may provide b with sufficient incentives to default. As a result, we cannot rule out that a lender ℓ with a belief μ_ℓ keeps his link to i at some state ω^0 only in equilibria in which i's strategy is to disconnect at time $t=0$ her link with one of the borrowers connected to her in an m-local monopolistic network G^0. In this sense, the set of LEBs may be too large – it allows for too many strategies, including some 'unreasonable' ones.

We now modify the definition of LEB (Definition 2) to require that a belief profile facilitates an equilibrium in which all lenders keep all of their links to intermediaries at $t=0$ *and* the strategies of all lenders and intermediaries in the network are invariant. In an invariant investment strategy, an intermediary's (lender's) action in an interaction with a borrower does not depend on the existence of other borrowers in her information set.

Definition 4. Consider any state ω with a network $G=\langle\langle L,I,B\rangle,E\rangle$, and an agent j that is either a lender or an intermediary $\left(j\in L\cup I\right)$. For any subset of borrowers $\widetilde{B}\subseteq B$, let $K_j|_{\widetilde{B}}(G)=K_j\left(G-\left(B-\widetilde{B}\right)\right)$ be the knowledge of j in a network $G-\left(B-\widetilde{B}\right)$ that consists of the subnetwork of G that includes exactly the agents $\left\langle L,I,\widetilde{B}\right\rangle$ and any link in G that connects two agents in $\left\langle L,I,\widetilde{B}\right\rangle$. A strategy σ_j is an *invariant investment strategy* if $\sigma_j\left(\omega'\right)=\sigma_j(\omega)\cap\left(I\cup\widetilde{B}\right)$ for any state ω' in which $K_j|_{\widetilde{B}}(G)=K_j\left(G\left(\omega'\right)\right)$ for some $\widetilde{B}\subseteq B$. A belief profile μ is an *Elementary Lending Enabling Belief profile* (ELEB) for network G if there exists an invariant investment strategy profile σ such that $\left(\sigma,\mu\right)$ is a pure strategy PBE-K^∞, and according to σ all lenders keep all of their links to intermediaries in G at time $t=0$ $\left(\forall_\ell\sigma_\ell\left(\omega^0(G)\right)\supseteq I_\ell^G\right)$. We denote by $LEB_E(G)$ the set of all ELEBs for network G. A network G is *LEB_E-robust* (or simply *robust*) if for any belief profile $\mu\in LEB_E(G)$ there exists a pure strategy PBE-K^∞ such that starting with $G^0=G$, the network G is sustained indefinitely $\left(\forall_{\mu\in LEB_E(G)}\exists_\sigma\left(\sigma,\mu\right)\in\Sigma(G)\right)$.[31]

An ELEB always exists. In fact, there exist a simple belief profile that is an ELEB for any network G.[32] We now present the main result of the paper.

Theorem 2. *A network is robust if and only if it is m-locally monopolistic.*

Theorem 2 highlights a major difference between the theory presented in this paper and the costly monitoring based explanation (e.g. Diamond 1984 and references therein) for the existence of intermediaries. Most of the costly monitoring literature provides a *local* theory of

[31]All of our analysis and results go through without any change also if the definition of robust networks required that the network be sustained indefinitely in a pure strategy PBE-K^∞ in which only invariant investment strategies are used.

[32]See Corollary 10 in the Appendix.

intermediation, i.e. the object of interest is a single investment opportunity consisting of *one borrower, one intermediary*, and a set of lenders with liquidity. As a result, a prediction of the costly monitoring literature is that the transaction goes through the intermediary. There may be predictions on the number of lenders that make an intermediary a viable option, but there is no prediction on the global patterns of intermediation. I.e. applying the costly monitoring theory separately to different one-intermediary-markets can result in having multiple intermediaries in an economy, but there are no predictions regarding the interconnectivity of such an economy, or the exclusivity of financial connections.[33] In contrast, our theory considers the entire financial network (over time) as the object of interest. As a result, we provide predictions on the global and intertemporal structure of financial networks, including intermediation.

To demonstrate one of the ways in which our theory is global, let an *active intermediary* be an intermediary who is connected to at least one lender and one borrower, and for any real number x, denote by $\lfloor x \rfloor$ the largest integer that is not larger than x. A direct implication of Theorem 2 is an upper bound on the number of *active intermediaries* in the economy.

Corollary 2. *Consider an economy with* $|B|$ *borrowers and* $|L| \geq m$ *lenders. Then,*

1. *The number of active intermediaries in any robust network is at most* $|B| \cdot \left\lfloor \frac{|L|}{m} \right\rfloor$*; and*
2. *For any* $1 \leq n \leq |B| \cdot \left\lfloor \frac{|L|}{m} \right\rfloor$*, there exists a robust network with* n *active intermediaries.*

4. Comparative statics

The upper bound on the number of active intermediaries captures only a small part of the picture. Theorem 2 provides sharp predictions with respect to the structure of the financial network in terms of $m = m(\rho, \lambda, \phi, r, p)$. A larger m implies that intermediaries are required to have more 'locally monopolistic' market power in order to enforce repayment.

An examination of the definition of m (equation 2.2 on page 12) yields the observation that m is decreasing in the frequency of the arrival of investment opportunities and liquidity (λ), the probability that an investment succeeds (q), the returns on a successful investment (r), and in the borrowers' share of the profit from investment $(1-\phi)$. On the other hand, m is increasing in the borrowers' discount rate (ρ). Notably, m does not depend on the size of the market, as captured by $|B|$ and $|L|$.

Heterogeneity in parameters across lenders and borrowers can be added into the model in the obvious way and generate corresponding heterogeneity in m across borrowers and relative weights for monopoly over different investment paths. For example, if borrowers are entrepreneurs seeking to fund a risky project, adding heterogeneity generates the following additional predictions: [1] older or less established entrepreneurs, who may have shorter expected

[33] Exceptions include Babus (2010) who provides a network formation theory for financial markets. Babus predicts that the financial network takes the form of a star network, with a single intermediation who is connected to all other agents.

horizons as entrepreneurs (larger ρ) are trusted only by intermediaries with significant market power, whereas established young to mid-career entrepreneurs are trusted also by intermediaries with less market power; and [2] Entrepreneurs who are considered to have a higher expected rate of arrival of investment opportunities, or investment opportunities with higher expected returns (λ, or q and r respectively) are trusted by a wide range of intermediaries with respect to market power.

From a macroeconomic perspective, the model predicts that in times of economic booms (high λ, r, and potentially q) there is room for a large number of intermediaries and more competitive markets in which any single intermediary has less market power (captured by m). Similarly, in highly liquid markets (e.g. low interest rates) in which borrowers receive larger shares of the profit $(1-\phi)$, the model predicts a large number of intermediaries, each of whom has less market power. On the other hand, in times of economic downturns, the model predicts that a small number of intermediaries may serve to mitigate the crisis by preventing a multiplier effect driven by the loss of trust. This prediction may be intensified if the economic downturn is triggered by a liquidity crunch and increases the bargaining power of lenders (as captured by ϕ).

Finally, we note that as long as all intermediaries are m-local monopolies, our model has nothing to say about the optimal number of intermediaries in a market or their optimal size. Thus, our model provides only a lower bound on intermediaries' market power, and a corresponding upper bound on the number of intermediaries in the economy. This is true even if we sharpen our prediction and require that a financial network facilitate the (constrained) maximum total expected returns. One natural way to get even sharper predictions is to incorporate into our framework a model of market volatility, as captured by an explicit low probability of an agent going out of business, and characterize the set of total expected returns maximizing robust networks that minimize the expected (or maximal) loss following such a local collapse. Depending on the details of the stochastic process underlying the initial collapse of institutions, this new criteria may favor networks with many small intermediaries, or with few intermediaries holding significant market power. We leave this extension for future research.

5. Discussion

In this section we evaluate the theoretical plausibility and meaningfulness of our notion of robustness of financial networks. Less theoretically inclined readers may wish to skip to the analysis of self-financing, collateralized debt, and bankruptcy, as well as of credit bureaus, and of price competition in sections 6 - 8.

5.1. **Consistency.** A network is robust if and only if it is m-locally monopolistic. Thus, if we consider robustness to be a plausible criterion, one might expect agents to maintain beliefs that are biased towards m-locally monopolistic networks. The following result shows that the set of m-locally monopolistic networks is consistent – if all agents believe that they are in an m-locally monopolistic network, then any m-locally monopolistic network can be sustained in a pure strategy PBE-K^{∞}, and any non m-locally monopolistic networks cannot.

Let $\mathscr{M}^{m,LM}$ be the set of belief profiles such that $\mu \in \mathscr{M}^{m,LM}$ if and only if μ satisfies the following condition: for any agent $j \in L \cup I$, if μ_j puts positive probability on a network G, then all of the lenders and intermediaries in G (potentially apart from j) are m-local monopolies. The proof of Corollary 3 follows closely the proof of Theorem 2 and is omitted.

Corollary 3. *A network G is m-locally monopolistic if and only if for any belief profile $\mu \in \mathscr{M}^{m,LM}$ there exists a pure strategy PBE-K^{∞} such that starting with $G^0 = G$, the network G is sustained indefinitely with probability* 1 $\left(\forall_{\mu \in \mathscr{M}^{m,LM}} \exists_{\sigma} \left(\sigma, \mu\right) \in \Sigma\left(G\right)\right)$.

5.2. **Benchmark approaches.** In this section we verify that the robustness criterion provides a meaningful refinement relative to candidate benchmarks. There are two dimensions in which PBE-K^{∞} can be relaxed: first, agents can observe more of the network. Second, the robustness requirement itself can be relaxed.

5.2.1. *Intermediate levels of observability of financial connections.* In section 2.2 we explore a game in which financial connections are perfectly observable at all times. In this section, we highlight the monotonic effect of observability on the set of robust networks.

We say that a network G is K-robust if it is robust with respect to observability structure $K = \left\{K_j\right\}_{j \in V}$.[34] Then, the following observation shows that if we increase the amount of information that agents observe, the set of robust networks increases.

Corollary 4. *Consider a network G and two observability structures K and K' such that for any agent j, $K_j \subseteq K'_j$. Then, if G is K-robust, then it is also K' robust.*

The proof is simple, if more information is available, a smaller set of belief profiles needs to be considered, so it is more likely that a PBE-K in which G is sustained indefinitely exists for any belief from the smaller set.

However, changes to the observability structure may be more nuanced than the partial order considered in Corollary 4. In particular, an assumption that financial connections are perfectly observable at all time incorporates two separate assumptions: [1] the initial network G^0 is perfectly observable (i.e. financial accessibility is observable); and [2] any adjustment that any agent j does to her financial network (e.g. disconnecting a link jb) is perfectly observable by all agents in the market, independent of their position in the network relative to j.

Example 3 highlights that our informational assumptions provide a refinement also relative to a model in which the initial network of financial connections is perfectly observable, yet agents observe adjustments done to the network only by inference that is based on their financial interactions (as captured by K^{∞}).

[34]The formal definition simply requires substituting K^{∞} with K in Definitions 2 and 4.

Example 3. *Assume that the parameters of the model are such that* $m = 3$*. Therefore, the two networks in Figure 5.1 are not* m*-locally monopolistic. Therefore, they are not robust. On the other hand, we saw in section 2.2 that in a model in which agents have perfect knowledge of the network structure at all times, either network can be sustained in a subgame perfect equilibrium. Now consider the following intermediate level of observability: agents observe the initial network, but observe changes to the network structure as captured by* K^{∞}*. In this case, there is no PBE-*K^{∞} *in which network (a) in Figure 5.1 is sustained indefinitely. To see why, note that if* b *defaults on* i_1*, then* i_2*,* ℓ_3*, and* ℓ_4 *cannot observe the default or the disconnection of any of the links* (i_1, b)*,* (ℓ_1, i_1)*, or* (ℓ_2, i_1)*. Thus,* b *expects to lose access to liquidity from at most two lenders following the default.*

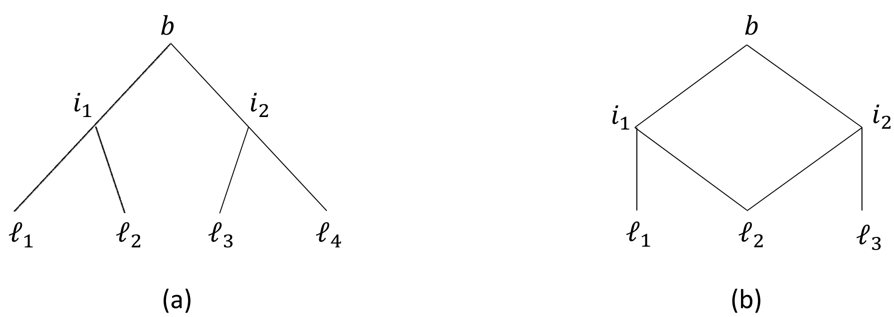

FIGURE 5.1.

On the other hand, consider network (b) in Figure 5.1. We claim that for any belief profile that is consistent with all agents knowing the initial network structure, there exists a PBE-K^{∞} *in which network (b) is sustained indefinitely with probability 1. The reason lies in the observation that it is common knowledge to* i_1*,* i_2*, and* b *that* i_1 *and* i_2 *are not* m*-local monopolies in network (b). Thus, if* b *defaults on* i_1*, then* i_2 *observes the disconnection of the link* (i_1, b) *and independently of the actions of* ℓ_1 *and* ℓ_2*, intermediary* i_2 *knows that she cannot enforce repayment by* b *any longer. In other words,* i_2 *observes* b*'s default on* i_1*, and has the incentives to react by disconnecting from* b*; and* i_1 *and* b *know that.*

5.2.2. *Robustness versus existence.* We are interested in characterizing networks that can be sustained indefinitely. In this section, we compare the following two sets: the set of robust networks, and the set of network for which there exists an equilibrium in which they are sustained indefinitely. We first note that Corollary 4's intuitively appealing monotonicity does not hold for the set of network for which there exists an equilibrium in which they are sustained indefinitely. To see why, note that increasing the amount of information observable has two effects: first, fewer belief profiles are admissible. Second, agents have an improved ability to coordinate on punishing a defaulting borrower. For the set of robust networks, both effects go the same way. However, for the set of network for which there exists an equilibrium in which they are sustained indefinitely, a smaller set of beliefs allows for fewer equilibria, which goes in the opposite direction from the positive effect of information on coordination.

We now turn to a more direct comparison of the two sets. We maintain our observability assumptions. However, instead of focusing on robust networks, we characterize networks for which there exists at least one PBE-K^∞ in which the network is sustained indefinitely. Proposition 2 suggests that robustness provides a significant refinement to this benchmark.

Proposition 2. *Consider a network G in which $|B| \geq 2$ and suppose that $G^0 = G$. Then, there exists a pure strategy PBE-K^∞ in which the network G is sustained indefinitely with probability 1 $\left(\exists_{\sigma,\mu}\left(\sigma,\mu\right) \in \Sigma\left(G\right)\right)$ if and only if each borrower is financially related to at least m lenders or to none $\left(\forall_{b|\ell I b^G}\left|S_b^G\right| \geq m\right)$.*

The proof of the *only if* part is trivial – the number of lenders that a borrower has access to puts an upper bound on the penalty for defaulting. On the other hand, the proof of the *if* part relies on a very special belief profile that is tailored to fit the underlying network G. In particular, in many cases the priors of all borrowers cannot put any significant probability on the real underlying network, i.e. the set of networks characterized is not consistent (in the sense defined in section 5.1). Consequently, we are reluctant to conclude that Proposition 2 should be considered as having predictive value. On the contrary, the proof reinforces our focus on robust networks that can be enforced given a large set of belief profiles, some of which are extremely simple, and employing very simple strategies.[35]

6. SELF-FINANCING, COLLATERALIZED DEBT, AND BANKRUPTCY

So far, we focused on the stylized case in which lenders take on the entire risk of the investment opportunity. However, in financial markets, lenders often require that a borrower pledge full or partial collateral, and bankruptcy rules make lenders the residual claimants of the firm's assets. It is also common that investors require that an entrepreneur self-finance a part of the investment opportunity from her personal funds. These measures often come with substantial efficiency costs. For example, liquidating a collateral may be costly,[36] and there is often a gap between the value of a firm's assets to the entrepreneur and the corresponding value to lenders (which is often simply the resale value of the assets).[37]

In this section, we evaluate the effect of the availability of self-financing, full and partial collateral, and bankruptcy laws, on the set of robust networks. We find that the aforementioned

[35]Naturally, we cannot rule out that a more intuitive proof exists. However, it cannot be based on a belief that is consistent with the true underlying network structure. Thus, consistency will still be violated. See also Example 4 on page 38.

[36]Pledging collateral may also block relatively liquid funds that can otherwise be invested at a positive return elsewhere.

[37]See also Bolton and Scharfstein (1996) and Kiyotaki and Moore (1997).

instruments do not change our results qualitatively. However, they do affect the level of monopoly power that an intermediary is required to have in order to enforce repayment. More specifically, allowing a borrower to self-finance a positive fraction of the investment opportunity reduces the monopoly power required by intermediaries in order to enforce repayment. The same is true for the availability of partial collateral and the presence of bankruptcy laws. On the other hand, introducing the possibility of pledging full collateral increases the monopoly power that intermediaries need in order to enforce repayment of uncollateralized investments. Therefore, if the cost of pledging full collateral is sufficiently low, the market may revert to simple debt contracts even when equity-like contracts are more efficient. Our analysis in this section requires extending Assumption 1 in the obvious way.

6.1. **Self-financing and riskless investments.** Investors commonly condition the provision of liquidity on a pre-agreed level of self-financing by an entrepreneur. In this section, we extend our model to allow for an industry norm, or a policy mandate, requiring a level of self-financing. We show that partial self-financing allows for the existence of smaller intermediaries with lower levels of monopoly power, and that all of our comparative statics extend immediately to this setup. Extending the model to allow for heterogeneous self-investment clauses is also discussed.

Consider the following contract with self-investment: suppose that lender ℓ has liquidity and borrower b has an investment opportunity. Then, ℓ invests a fraction $\psi \in (0,1)$ of liquidity to b under the condition that b invest the remainder $(1-\psi)$ from his personal funds. The details of the contract follow the ones suggested in section 2, scaled down by the factor ψ. I.e. if ℓ invests the money directly with b, and if the investment opportunity has a positive outcome $\left(\frac{1+r}{q}\right)$, then b is required to pay $\psi\left(\frac{1+\phi r}{q}\right)$ to ℓ. Similarly, if ℓ invests the money via an intermediary i who invests it with borrower b, and if the investment opportunity has a positive outcome then b is required to pay $\psi\left(\frac{1+\phi r}{q}\right)$ to i who upon receiving the payment is required to pay $\psi\left(\frac{1+\phi r-r_I}{q}\right)$ to ℓ, and keep to herself an intermediation fee of $\frac{\psi r_I}{q}$.[38] Let χ be such that $1>\chi>\phi$ and assume that a borrower has a cost of $(1-\psi)\chi r$ per $1-\psi$ of self-financing (e.g. cost of liquidating illiquid assets), and that borrowers never find it profitable (or feasible) to take the investment opportunity with 100 percent self-financing – e.g. due to limited personal wealth or need for diversification (allowing borrowers to fully self-finance at some cost is qualitatively identical to the case in which borrowers have an illiquid asset that can be used as full collateral with some cost of liquidation – see section 6 below).

We say that a network is (ψ,χ)-robust if it is robust given that contracts have $1-\psi$ self-financing at cost $(1-\psi)\chi r$. The following Corollary is a direct implications of Theorem 2.

[38]Liquidity constraints and lenders' incentive constraints imply that $\phi \in \left[-\frac{1}{r},1\right]$, and $r_I \in [0,\phi r]$.

Corollary 5. *Let* $m^{\psi,\chi} = min\left\{n \in \mathbb{N} | \frac{1}{\rho} \cdot n \cdot \lambda \cdot [\psi(1-\phi) + (1-\chi)(1-\psi)]\, r \geq \psi\left(\frac{1+\phi r}{p}\right)\right\}$. *Then, a network is* (ψ, χ)*-robust if and only if it is* $m^{\psi,\chi}$*-locally monopolistic.*

Corollary 5 suggest that self-financing allows for the existence of smaller intermediaries with lower levels of monopoly power, and that all of our results and comparative statics extend immediately to this case with the only change that m is substituted with $m^{\psi,\chi}$. Moreover, Corollary 5 can be extended to include investments with zero self-financing, but with a fraction that is riskless, i.e. a fraction of the return on investment is certain and therefore repayment of this portion is fully contractible.

Extending our analysis to endogenously determined fractions of self-financing require additional structure. For example, why would a borrower prefer not to self finance? If the reason is limited liquid balance then a borrower always prefers to self-finance the largest fraction possible given her constraint. On the other hand, if there is an efficiency cost for using self-financing, e.g. cost of liquidating illiquid assets, then under some specifications a borrower prefers the lowest fraction of self-financing. Nevertheless, regardless of the particular structure imposed, it is the case that intermediaries who have higher levels of monopoly power can enforce repayment with a wider range of fractions of self-financing.

6.2. **Collateralized debt and bankruptcy.** An additional widely used instrument for preventing strategic default is the use of a firm's (potentially illiquid) assets as collateral. This can be done explicitly by assigning a collateral to the loan itself, or implicitly by endowing lenders with the power to compel the firm to announce bankruptcy and distribute its assets between stake holders.

Assume that at any point in time, each borrower has one illiquid asset, with a discounted present value of V_B to the borrower and V_L to the lender. Following Kiyotaki and Moore (1997), we assume that $V_B > V_L$. Thus, using a collateral is costly and generates an efficiency loss of $V_B - V_L$ whenever the collateral is transferred to the lender.

Consider the following contract: suppose that lender ℓ has liquidity and borrower b has an investment opportunity. Then, if ℓ invests directly with b, and if the investment opportunity has a negative outcome, then b is required to give the collateral to ℓ, whereas if the investment opportunity has a positive outcome $\left(\frac{1+r}{q}\right)$, then b is required to pay $\frac{1+\phi r-(1-q)V_L}{q}$ to ℓ. If ℓ invests via an intermediary i who invests with borrower b, and if the investment opportunity has a negative outcome, then b is required to give the collateral to ℓ, whereas if the investment opportunity has a positive outcome, then b is required to pay $\frac{1+\phi r-(1-q)V_L}{q}$ to i who upon receiving the payment keeps to herself an intermediation fee of $\frac{r_I}{q}$ and transfers the remainder to ℓ.[39]

[39]To see how the repayment sum upon successful investment $\left(\frac{1+\phi r-(1-q)V_L}{q}\right)$ is calculated, note that $p\frac{1+\phi r-(1-p)V_L}{p} + (1-p)V_L = 1+\phi r$. Thus, conditional on not strategically defaulting, the expected payment by a borrower did not change under this specification relative to the main analysis of this paper. Allowing lenders and

We say that a network is (V_B, V_L)-robust if it is robust with respect to contracts that incorporate an asset parametrized by V_B and V_L. The following Corollary is a direct implications of Theorem 2.

Corollary 6. *Let* $m^{V_B,V_L} = min\left\{n \in \mathbb{N} | \frac{1}{\rho} \cdot n \cdot \lambda \cdot \left(\left(1-\phi\right) r + \left(1-q\right)(V_L - V_B)\right) \geq \frac{1+\phi r-(1-q)V_L}{q}\right\}$*. Then, a network is* (V_B, V_L)*-robust if and only if it is* m^{V_B,V_L}*-locally monopolistic.*

Corollary 6 suggests using collateral enables a larger set of networks to be robust if and only if $m > m^{V_B,V_L}$. On the other hand, if $m \leq m^{V_B,V_L}$ then using a collateral has efficiency cost with no additional benefits in terms of enforcement. The reason is that the use of a collateral affects both the immediate benefit of a borrower from strategic default, and the expected future payoffs of a borrower that expects to be asked for a collateral in all future transactions. Evaluating the expressions for m and m^{V_B,V_L}, we conclude that a collateral enhances investment when V_L is large and $V_B - V_L$ is small. That is, a collateral enhances investment when the collateral is valuable and induces little expected efficiency loss in any given transaction. The following simple cases provide additional intuition:

(1) If $V_B = V_L$ then a collateral always allows for intermediaries with (weakly) lower levels of monopoly power.
(2) If $V_L \geq 1+\phi r$ and $V_B - V_L < \frac{(1-\phi)r}{1-q}$, then a collateral allows for direct investment between any lender and borrower.

In the latter case, fully collateralized debt contracts are available. However, as long as $V_B > V_L$, collateralized debt contracts come with an efficiency loss. Thus, it raises an additional question: how does the availability of fully collateralized debt contracts affect the market's ability to enforce efficient investment contracts that require no collateral? This question is related to another question that has longer history in the financial literature, namely: which markets do we expect to transact mostly on collateral, and which markets do we expect to transact based on long term relationships (potentially using intermediaries for enforcement).[40]

6.2.1. *Collateral can be liquidated to generate* $> 1 + \phi r$ *at a cost.* In this section we focus on the case where $V_L \geq 1 + \phi r$ and $0 < V_B - V_L < \frac{(1-\phi)r}{1-q}$, so that fully collateralized debt contracts are available but carry an efficiency cost, and answer the following question: what networks can sustain repayment in a robust and efficient (i.e. no collateral) manner?

In the presence of fully collateralized contracts, any lender agrees to invest with any borrower using a collateral. Thus, any borrower who is refused an uncollateralized investment via the

intermediaries to demand lower shares due to the reduction in uncertainty does not change our analysis as long as it does not eliminate completely the expected loss to the borrower from needing to liquidate an asset worth V_B in order to pay V_L.

[40]See also Babus (2010) and references therein.

network, can turn to the entire pool of lenders and publicly announce that he offers a fully collateralized investment opportunity. This increases the outside option of a borrower and makes it more difficult to enforce the repayment of investments made with no collateral.

Corollary 7. *Let* $\bar{m} = min\left\{n \in \mathbb{N} | \frac{1}{\rho} \cdot n \cdot \lambda \cdot \left(1-q\right)(V_B - V_L) \geq \frac{1+\phi r}{q}\right\}$*. Then, a network is robust if and only if it is* $\bar{m}$*-locally monopolistic.*[41]

Corollary 7 suggests that reducing the cost of pledging a collateral $(V_B - V_L)$ has two effects: on the one hand, it makes fully collateralized investments more efficient. On the other hand, it makes it more difficult to enforce efficient uncollateralized investments.

In addition, Corollary 7 shows that the ability to transact without collateral can be traced back to the riskiness of the underlying investment opportunity. In particular, holding fixed the expected return on investment $(1+r)$, a higher probability that the investment succeeds implies that the expected cost of pledging a collateral $\left(1-q\right)(V_B - V_L)$ is lower, which makes it more difficult to enforce repayment without collateral. At the same time, a higher probability that the investment succeeds implies that the expected temptation to default $\left(\frac{1+\phi r}{q}\right)$ is lower, which makes it easier to enforce repayment without collateral. combining these two effects we get that when full collateral is available, **the riskiest and safest investments are expected to be traded using collateral, whereas the intermediately risky assets are traded by intermediaries and without collateral**.

7. Credit bureaus and generalized central information entities

Standard credit rating agencies and bureaus have no explicit role in the setting considered in this paper. Credit bureaus generally rate the *ability of a debitor to repay*, rather than *the incentives of a borrower or entrepreneur to repay* when an investment succeeds and when strategic default is not verifiable. That is, credit bureaus focus on enabling creditors to avoid suffering from *liquidity defaults*, rather than from *strategic defaults*.

In this section we go beyond the standard credit bureaus and credit rating agencies. We consider (hypothetical) Generalized Central Information Entities (GCIEs) and their effect on the set of robust networks. We show that given 'reasonable' capabilities, a GCIE can relax the constraints imposed by the m-local monopolism requirement. However, as long as the frequencies of arrival of liquidity and investment opportunities are sufficiently low (so that $m > 1$), it is still the case that in all robust networks any lender and borrower that are financially related must also be connected to at least one intermediary in common. Also, any intermediary must be connected to at least m lenders or to none $\left(\forall_{\ell i b^G} \left|L_i^G\right| \geq m\right)$.

Clearly, if a GCIE can observe the entire network of financial connections instantaneously at any moment in time, it can facilitate ostracism and the result captured by Proposition 1 is

[41] The expression for $\bar{m}$ is an algebraically simplified representation of $\bar{m} = min\left\{n \in \mathbb{N} | \frac{1}{\rho} \cdot n \cdot \lambda \cdot \left(1-\phi\right) r \geq \frac{1+\phi r}{q} + \frac{1}{\rho} \cdot n \cdot \lambda \cdot \left(q\frac{1+r}{q} + \left(1-q\right)(V_L - V_B) - \left(1+\phi r\right)\right)\right\}$.

recovered: *a network G is robust if and only if each borrower is financially related to at least m lenders or to none* $\left(\forall_{b|\ell I b^G} \left|S_b^G\right| \geq m\right)$. To enforce repayment, the perfectly informed GCIE gives the highest rating to any borrower who is financially related to at least m lenders and never strategically defaulted in the past, and the lowest rating to any other borrower. This facilitates a bang-bang strategy profile as described in section 2.2 – a single strategic default by one borrower leads to the elimination of all of the links in the network.

On the other extreme, if a GCIE cannot observe the network structure, and cannot verify that a lack of repayment is indeed a strategic default, then our main result (Theorem 2) goes through without change: *the set of robust networks and the set of m-locally monopolistic networks are identical.*

The more interesting case is a GCIE that can perfectly observe a strategic default whenever such a default occurs, yet does not have the information to map the financial network reliably. Assume that such a GCIE exists, and that whenever any borrower b strategically defaults the GCIE announces the identity of the defaulting borrower. Finally, in accordance with the spirit of our model, any announcement of a default triggers a network adjustments stage in which each lender and intermediary can decide which of their links they keep and which they eliminate. Then, Proposition 3 offers a partial characterization of the set of robust networks.

Proposition 3. *Consider the model with a GCIE that publicly announces the identity of any borrower who defaults, and assume that the parameters of the model are such that* $m > 1$. *Then, a network G is robust only if*

[1] If a lender ℓ *is connected directly to a borrower b* $\left(\ell b^G\right)$, *then there exists an intermediary i who is connected to both* ℓ *and b; and*

[2] Every intermediary i who is connected to at least one lender and one borrower is connected to at least m distinct lenders in G $\left(\forall_{\ell i b^G} \left|L_i^G\right| \geq m\right)$.

Proposition 3 establishes that even in a model with a GCIE who publicly announces the identity of any borrower who ever defaulted, robustness requires that intermediaries exist and be well connected.

8. EXTENSION: PRICING AND MARKET STRUCTURE

In this section we explore the connection between the details of the investment contracts with respect to prices (i.e. shares of returns), and the set of robust financial networks. First, we verify that our results are *not* artifacts of the fixed price contracts; they carry over to an environment in which intermediaries compete in prices à la Bertrand for each investment. Second, we show that when the division of surplus between lenders and borrowers follows the Nash bargaining solution, the set of robust financial networks may include also networks in which intermediaries have sufficient duopolistic market power.[42]

8.1. **Price competition among intermediaries.** In this section we verify that Theorem 2 extends to an environment in which intermediaries compete in prices à la Bertrand. Formally, suppose that when an intermediary i who is connected to both ℓ and b informs both that she is able to intermediate the transaction (stage 1-(b) in section 2.1), i also chooses a fee $\frac{r_i(h_i)}{q}$ for the transaction, where $r_i(h_i) \in [c, r_{max}]$ for some $r_{max} < \phi r$. As before, ℓ and b choose uniformly at random one of the investment paths that maximizes their combined payoff (i.e. if ℓ and b are not connected directly, they pick randomly one of the intermediaries who charge the minimal fee).[43]

A *pure strategy Bertrand PBE-K^∞* is a pure strategy PBE-K^∞ in which intermediaries choose prices in the following way. Suppose that at time t lender ℓ has liquidity and borrower b has an investment opportunity. Suppose further that intermediary i is connected to both ℓ and b. Then, i determines her fee as follows:

$$r_i(h_i) = \begin{cases} c & \text{if there exists an intermediary } i' \neq i \text{ such that } \ell i' b^G \\ r_{max} & \text{otherwise} \end{cases}. \tag{8.1}$$

A network G is *Bertrand-robust* if for any belief profile $\mu \in LEB_E(G)$ there exists a pure strategy Bertrand PBE-K^∞ such that starting with $G^0 = G$, the network G is sustained indefinitely with probability 1. The proof of Corollary 8 follows the same argument as the proof of Theorem 2 and is omitted.

Corollary 8. *A network is Bertrand-robust if and only if it is m-locally monopolistic.*

Corollary 8 can be replicated with additional notions of competition and bargaining. However, generalizing Corollary 8 to allow for intertemporal competition and bargaining solution concepts presents technical complications that go beyond the scope of this paper.[44]

[42]The division of surplus between a lender and a borrower must be based on a bargaining solution concept rather than price competition. This is because at any point in time, the probability that more than one lender has liquidity and/or more than on borrower has an investment opportunity is zero.

[43]We define $r_i(h_i)$ to belong to a closed set to allow for the existence of an equilibrium in which intermediaries compete in prices à la Bertrand. The additional restriction that $r_{max} < \phi r$ allows us to abstract from indifferences on the lenders' side of the market. An alternative approach that yields identical results without restricting $r_i(h_i)$ and r_{max} is to model money using a discrete variable.

[44]The main challenge is to consider the effect that price competition has on agents' knowledge of the financial network that they are embedded in – i.e. K^∞. Pricing can affect K^∞ in two ways. First, different pricing schemes may induce different frequencies of transactions for two intermediaries that connect a certain lender-borrower pair. This may distort the knowledge of the network that intermediaries have. Second, intermediaries may use prices to communicate private information to lenders and borrowers.

8.2. **Bargaining and local duopolies.** We now show that considering richer bargaining outcomes between lenders and borrowers has the potential to enrich our results in an interesting way: if we allow lenders and borrowers to divide their surplus according to a generalized Nash bargaining solution, we cannot rule out that the set of robust networks includes networks in which some intermediaries have sufficient *duopolistic* market power, even if they lack any monopolistic market power.

Formally, suppose that any intermediary i who is connected to both lender ℓ and borrower b informs them that she is able to intermediate the transaction and declares a fee $\frac{r_i(h_i)}{q}$. Then, ℓ and b choose an investment path as before, and decide on the terms of investment: $\phi_{\ell b}(h_\ell, h_b)$ for a direct investment, and $\phi_{\ell i b}(h_\ell, h_b, r_i)$ for an intermediated investment.

Consider a lender ℓ and a borrower b that decide how to divide their surplus. A generalized Nash bargaining solution requires that there is some $\bar{\phi}$ such that the borrower pays to the intermediary

$$\frac{1+\bar{\phi}(r-r_i)+r_i}{q}, \tag{8.2}$$

and the intermediary transfers to the lender

$$\frac{1+\bar{\phi}(r-r_i)}{q}. \tag{8.3}$$

If the investment is made directly, then the same applies with a (hypothetical) fee of $r_i = 0$.

A *pure strategy Bertrand PBE-K^∞ with Nash bargaining parametrized by* $\bar{\phi}$ is a pure strategy PBE-K^∞ in which any intermediary determines her fee according to (8.1), and repayment is done according to (8.2) and (8.3) for some $\bar{\phi}$ such that $\bar{\phi}(r-r_{max})+r_{max} \le \phi_{max} r$. A network G is *Bertrand-robust with Nash bargaining parametrized by* $\bar{\phi}$ if for any belief profile $\mu \in LEB_E(G)$ there exists a pure strategy Bertrand PBE-K^∞ with Nash bargaining parametrized by $\bar{\phi}$ such that starting with $G^0 = G$, the network G is sustained indefinitely.

With a slight modification of m to incorporate $\bar{\phi}$ and r_{max} instead of ϕ in Equation (2.2) it is still true that m-locally monopolistic networks are Bertrand-robust with Nash bargaining parametrized by $\bar{\phi}$. In addition, since a borrower b pays lower intermediation fees when he has more than one investment path to a given lender, b might also sufficiently value a relationship with an intermediary who has sufficient duopolistic market power – an intermediary who is a part of a large number of investment paths; each being one of exactly two intermediated paths connecting a lender and b. Formally, let

$$
\begin{aligned}
M^D &= M^D\left(\rho,\lambda,r,p,c,\bar{\phi},r_{max}\right)\\
&= \left\{\left(n^M,n^D\right)\in\mathbb{N}^+\times\mathbb{N}\mid \frac{1}{\rho}\cdot n^M\cdot\lambda\cdot\left(1-\bar{\phi}\right)(r-r_{max})+\right.\\
&\left.\frac{1}{\rho}\cdot n^D\cdot\lambda\cdot\left(1-\bar{\phi}\right)(r_{max}-c)\geq\frac{1+\bar{\phi}(r-r_{max})+r_{max}}{q}\right\}\cup\\
&\cup\left\{\left(0,n^D\right)\in\{0\}\times\mathbb{N}\mid\frac{1}{\rho}\cdot n^D\cdot\lambda\cdot\left(1-\bar{\phi}\right)(r_{max}-c)\geq\frac{1+\bar{\phi}(r-c)+c}{q}\right\}.
\end{aligned}
\tag{8.4}
$$

We say that an intermediary i is a $\left(k^M,k^D\right)$-*local duopoly* in G if for any investment path $\ell i b$ in G there exist at least k^M lenders whose only investment path to b passes via i, and at least k^D lenders whose one of exactly two intermediated investment paths to b passes via i. A network G is M^D-*locally monopolistic* if any intermediary in G is a $\left(k^M,k^D\right)$-local duopoly in G for some $\left(k^M,k^D\right)\in M^D$. Then, the proof of the following partial characterization result follows the same argument as Theorem 2 and is omitted.[45]

Proposition 4. *Let $\frac{1}{\rho}\cdot\lambda\cdot\left(1-\bar{\phi}\right)r<\frac{1+\bar{\phi}r}{q}$. Then, a network is Bertrand-robust with Nash bargaining parametrized by $\bar{\phi}$ only if it is M^D-locally duopolistic and has no direct links between lenders and borrowers.*[46]

9. Conclusion

This paper proposes a theory of intermediation: an intermediary who exclusively represents a large pool of lenders in their transactions with a borrower can enforce repayment by the borrower even when the frequency of interactions of the borrower with each individual lender is low.

We show that incomplete knowledge of the network of investment patterns is key for understanding the role of intermediation in the informal enforcement of lending and investment contracts. Imperfect observability of financial connections may prevent lenders and intermediaries from relying on community enforcement via ostracism and contagion to provide the necessary incentives for repayment. Therefore, only intermediaries with sufficient market power (as captured by the notion of m-local monopoly) can be certain that they can execute financial transactions successfully, and intermediation can lead to a welfare improvement by allowing for the execution of transactions that would not have been possible otherwise. This is true even if intermediation is costly.

[45]For the proof one needs to update Assumption 1 with r_{max} and $\bar{\phi}$ in the obvious way.

[46]The focus on parameters such that $\frac{1}{\rho}\cdot\lambda\cdot\left(1-\bar{\phi}\right)r<\frac{1+\bar{\phi}r}{q}$ restricts our attention to environments in which direct links between lenders and borrowers are not sustainable. It is done for notational convenience only – a similar result with additional notation can be derived if $\frac{1}{\rho}\cdot\lambda\cdot\left(1-\bar{\phi}\right)r\geq\frac{1+\bar{\phi}r}{q}$.

By emphasizing exclusivity and market power, our analysis suggests that the size of an intermediary (as measured by her connectivity to lenders and borrowers) matters. However, it is not a sufficient statistic to determine whether an intermediary can enforce cooperation. An effective intermediary must be of a size that is above some threshold and at the same time, for any borrower that the intermediary transacts with, the intermediary must exclusively represents a sufficiently large pool of lenders in their transactions with that borrower. In that sense our model of intermediation is global – rather than focusing on a single transaction and a single intermediary, we characterize networks that are robust and show that in any such network all intermediaries have sufficient market power. The minimal market power required of an intermediary depends on the fundamental of the economy. If the frequency of arrival of investment opportunities and the expected returns on investments are high, intermediaries may enforce repayment without holding significant market power. However, if investment opportunities are scarce and carry low expected returns, only large intermediaries who hold significant market power are effective.

We also find that self-financing clauses in contracts and partial collaterals complement the role of the intermediary in enforcing repayment and allow intermediaries with lesser market power enforce repayment. On the other hand, the possibility of pledging full collaterals make it more difficult to enforce repayment of non-collateralized investments. If collateral is more costly than intermediation, this can lead to an efficiency loss. Finally, we explore the role of credit rating agencies, and show that more general credit information agencies may make the enforcement problem easier, but not eliminate the scope for intermediation.

APPENDIX

Definition 5. Suppose that in the game described in section 2.1 the network $G\left(\omega^t\right)$ is sustained indefinitely starting from time t. Consider an agent $j \in G(\omega)$ and let $\mu_j \in \mathscr{M}_j$ if belief μ_j satisfies the following conditions:

[1] If j is a lender then $Pr_{\mu_j^t}\left(\langle I_j, B_j\rangle = \left\langle I_j^{G(\omega^t)}, B_j^{G(\omega^t)}\right\rangle\right) = 1$ and if j is an intermediary $Pr_{\mu_j^t}\left(B_j = B_j^{G(\omega^t)}\right) = 1$ – i.e. j knows who she trusts;

[2] $Pr_{\mu_j^t}\left(\omega = \omega^t\right) > 0$ – i.e. at time t, agent j puts positive probability on the true state;

[3] $\forall_{t'>t} Pr_{\mu_j^{t'}}\left(G^{t'} = G\left(\omega^t\right)\right) = 1$ – j puts probability 1 on the network not changing anytime after time t; and

[4] μ_j is consistent with Bayes law in the induced game described in section 2.1 starting with time t, with the additional restriction that the update following any event that occurs at any

time t' must also be consistent with Bayes rule following information of the same event happening during some arbitrarily small positive time interval that includes t' instead of exactly at t'.[47]

In this setup, we denote by $K_j^{T,\epsilon}\left(G\left(\omega^t,\mu_j\right)\right)$ the set of networks such that $G' \in K_j^{T,\epsilon}\left(G\left(\omega^t,\mu_j\right)\right)$ if at time $T > t$ the network G' is assigned a probability larger than ϵ by μ_j $\left(K_j^{T,\epsilon}\left(G\left(\omega^t,\mu_j\right)\right) = \left\{G' | Pr_{\mu_j}\left(G^T = G'\right) > \epsilon\right\}\right)$, and let $K_j^{\infty}\left(G\left(\omega^t\right)\right) = \left\{G' | \exists_{\mu_j \in \mathscr{M}_j} \exists_{\epsilon>0} G' \in lim_{T\to\infty} K_j^{T,\epsilon}\left(G\left(\omega^t,\mu_j\right)\right)\right\}$ be the set of *steady state believable networks* of agent j. We say that a network G' is a *steady state believable network* if it is a steady state believable network for all agents.

Proposition 5. *The following two statements are equivalent.*

[1] $G' = \left\langle\left\langle B', I', L'\right\rangle, E'\right\rangle$ is a steady state believable network.

[2] All of the following hold:

2a.$B' = B\left(\omega^t\right)$;

2b.If $j \in L$, then $I'_j = I_j\left(\omega^t\right)$ and $B'_i = B_i\left(\omega^t\right)$ for all $i \in I_j$;

2c.If $j \in I$, then $L'_j = L_j\left(\omega^t\right)$, $B'_j = B_j\left(\omega^t\right)$, plus for each $\ell \in L_j$ and $b \in B_j$ such that $\ell j b^{G(\omega^t)}$ it is the case that $\ell b^{G'}$ if and only if $\ell b^{G(\omega^t)}$ and if $\neg \ell b^{G(\omega^t)}$ then $\left|\left\{i' \in I | \ell i' b^{G(\omega^t)}\right\}\right| = \left|\left\{i' \in I | \ell i' b^{G'}\right\}\right|$; and

2d.If $j \in B$, then $I'_j = I_j\left(\omega^t\right)$ and $L'_i = L_i\left(\omega^t\right)$ for any $i \in I_j$.

Proof of Proposition 5. **[1]⇒[2]:** Consider a network $G' = \left\langle\left\langle B', I', L'\right\rangle, E'\right\rangle$ such that one of the conditions 2a-2d does not hold. Then it is left to show that for any belief μ_j that is consistent with the requirements of Definition 5, $G' \notin K_j^{\infty}\left(G\left(\omega^t\right)\right)$. Namely, for any $\epsilon > 0$ there exists $t' > t$ such that $Pr_{\mu_j}\left(G^{t'} = G'\right) < \epsilon$. For 2a, 2b, and 2d, as demonstrated in Example 1, as $t' \to \infty$ the probability of an event after which $Pr_{\mu_j}\left(G^{t'} = G'\right) = 0$ goes to one. Finally, as demonstrated in Example 1, an application of the law of large numbers suggests that the probability of the network being a network that violates 2c goes asymptotically to zero as t' goes to infinity.

[2]⇒[1]: Consider a belief μ_j that is consistent with the requirements of Definition 5 such that $\mu_j\left(\omega^t\right)$ puts probability p on G' and probability $1-p$ on the real network $G\left(\omega^t\right)$, i.e. $\forall_{t' \geq t} Pr_{\mu_j}\left(G = G'\right) = p$ and $\forall_{t' \geq t} Pr_{\mu_j}\left(G = G\left(\omega^{t'}\right)\right) = 1-p$. It is left to verify that for any $t' > t$ and any history of play possible during $\left[t, t'\right]$ according to the game described in section 2.1 (and given that the network is sustained during $\left[t, t'\right]$), the probability that the history is observed by j given G' equals the probability that the same history is observed by j given $G\left(\omega^t\right)$ during , the same time period. The verification follows the same steps as in Example 1 and is omitted.■

Example 4. *Consider network (a) in Figure 9.1 and assume that the parameters of the model are such that $m = 4$. If network (a) is the initial network $\left(G^0\right)$ and if the beliefs of all agents put*

[47]The additional restriction alleviates the difficulty of applying Bayes rule in a stochastic continuous time model (in which trade opportunities arrive according to a Poisson arrival process). The problem rises because the probability of an investment opportunity at any given time t' is zero. The restriction simply implies that when an event is considered a zero probability event because of the specific time it occurs, an agent cannot completely ignore the information in the even in her update. Moreover, in a discrete version of our model the additional restriction can be removed.

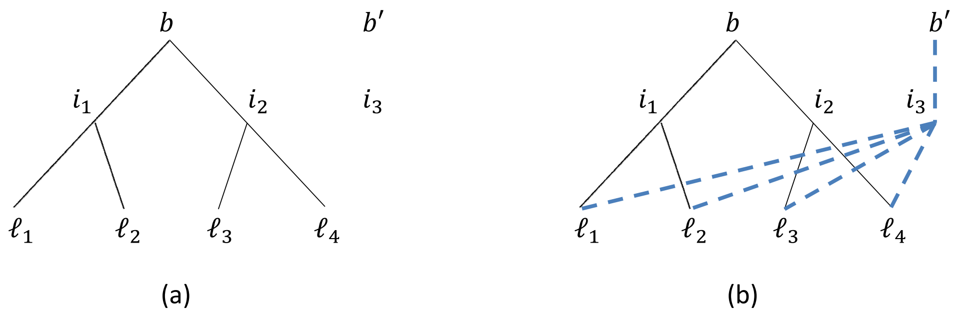

FIGURE 9.1.

probability 1 on the correct network, then there is no pure strategy profile such that the network (a) is sustained indefinitely in a pure strategy PBE-K$^\infty$. To see why, note that if b defaults on an investment agreement intermediated by i_1, then the only agents who learn about the default (either directly, or by observing subsequent changes to the network) are i_1, ℓ_1, and ℓ_2.

Nevertheless, there exists a belief profile that is part of a pure strategy PBE-K$^\infty$ in which the network (a) is sustained indefinitely with probability 1: assume that the belief of borrower b puts probability 1 on the network being network (b) in Figure 9.1, and assume that the priors of all other agents in the network put probability $\frac{1}{2}$ on network (a) and probability $\frac{1}{2}$ on network (b). Thus, at time $t = 0$ all lenders ($\ell_1 - \ell_4$) update their beliefs to put probability 1 on the correct network. Then, with the aforementioned belief profile, the following strategy profile constitutes a pure strategy PBE-K$^\infty$ in which network (a) is sustained indefinitely: [1] any agent j who is a lender or an intermediary keeps all of her links unless she observes a default or a disconnection of a link; [2] any borrower who believes that the network is network (b) does not default unless she observes a default or a disconnection of a link; and [3] any borrower who believes that the network is (a) always defaults.

Fact 1. $\forall_{G,\ell i b^G,\omega\in\Omega(G)}$,
$Pr\left(\left|S_b^G\right| - \left|S_b^{G-ib}\right| \geq m \mid h_i\right) = Pr\left(\left|S_b^G\right| - \left|S_b^{G-ib}\right| \geq m \mid h_b\right) = Pr\left(\left|S_b^G\right| - \left|S_b^{G-ib}\right| \geq m \mid G\right) \in \{0,1\}$.

Following Fact 1 and for $j \in \{i, b\}$ we often substitute
$\mu_j\left(\left|S_b^G\right| - \left|S_b^{G-ib}\right| \geq m\right) = 1$ $\left(\mu_j\left(\left|S_b^G\right| - \left|S_b^{G-ib}\right| < m\right) = 1\right)$ with the shorter $\left|S_b^G\right| - \left|S_b^{G-ib}\right| \geq m$ $\left(\left|S_b^G\right| - \left|S_b^{G-ib}\right| < m\right)$.

Proof of Lemma 1. Let $\bar{\mu}$ be any belief profile that is consistent with the following conditions for all G:

(1) For any $j \in I \cup L$, $\bar{\mu}_j$ puts probability 1 on a network G such that $\forall_{i'b^G | i' \neq j} \left|S_b^G\right| - \left|S_b^{G-i'b}\right| \geq m$.
(2) For any borrower b, $\bar{\mu}_b$ puts probability 1 on a network G such that $\ell i b^G \wedge \ell i b'^G \Rightarrow b = b'$ (this condition is redundant in the one borrower case).

Let $\bar{\sigma}$ be a strategy profile that is consistent with the following conditions for all G and $\ell i b^G$:

(1) For any $j \in L \cup I$, borrower b repays an investment on jb if $\left|S_b^G\right| - \left|S_b^{G-jb}\right| \geq m$.

(2) A borrower b does not repay an investment on ib if $\left|S_b^G\right| - \left|S_b^{G-ib}\right| < m$ and $\forall_{i'b \mid i' \neq i} \left|S_b^G\right| - \left|S_b^{G-i'b}\right| \geq m$.
(3) An intermediary i maintains her link to a borrower b if and only if $\left|S_b^G\right| - \left|S_b^{G-ib}\right| \geq m$.
(4) A lender ℓ maintains all of her links to intermediaries.
(5) A lender ℓ maintains her link to a borrower b if and only if $m = 1$ and there is no intermediary who is connected to both ℓ and b.

Note that for any network G' and two paths $\ell i b^{G'}$ and $\ell' i' b'^{G'}$ such that $ib \neq i'b'$ and for a network $G'' = G' - ib$, $\left|S_b^{G'}\right| - \left|S_b^{G'-i'b'}\right| \geq m$ only if $\left|S_b^{G''}\right| - \left|S_b^{G''-i'b'}\right| \geq m$. Thus, given $\bar{\sigma}$ an intermediary i who trusts a borrower b at $t = 0$, continues trusting borrower b as long as b does not default on i and as long as no lender ℓ disconnected from i. Given that, borrowers' strategies are best response. Similarly, intermediaries and lenders' strategies are best response given their beliefs and the strategies of all other agents.■

Let $\bar{L}_i^G \equiv \left\{\ell \in L_i^G \mid \ell i' b^G \Rightarrow i' = i\right\}$ be the set of lenders whose only investment path to b in network G is via i.

Lemma 2. *Consider an economy with one borrower. Consider any network G, intermediary i, and state ω' such that $G^0 = G$, $\bar{L}_i^G \subseteq L_i^{G(\omega')}$, and $\ell' i b^{G(\omega')}$ for at least one lender ℓ'. Consider any assessment (σ, μ) that is a pure strategy PBE-K^∞ and according to σ all lenders keep all of their links to intermediaries in G at time 0 $\left(\forall_\ell \sigma_\ell\left(\omega^0(G)\right) \supseteq I_\ell^G\right)$. Then, σ satisfies the following. If i is an m-local monopoly in G then in state ω':*
[1] i keeps her link to b $\left(b \in \sigma_i\left(h_i\left(\omega'\right)\right)\right)$; and
[2] b does not default on i $\left(i \in \sigma_b\left(h_b\left(\omega'\right)\right)\right)$.

Proof of Lemma 2. For any $i \in I$ and state ω' that satisfies the conditions of the Lemma, let $O_i^I\left(\omega'\right) = min_{\omega \mid K_i^\infty(\omega) = K_i^\infty(\omega')} \left|\left\{(\ell, i') \mid i' \in I - i, \ell i b^{G(\omega)}, \ell i' b^{G(\omega)}\right\}\right|$ and let $O_i^L\left(\omega'\right) = \left|\left\{\ell \mid \ell i b^{G(\omega')}, \ell b^{G(\omega')}\right\}\right|$. Note that by definition $O_i^I\left(\omega'\right)$ and $O_i^L\left(\omega'\right)$ can be perfectly inferred from $K_b\left(\omega'\right)$ as well as from $K_i\left(\omega'\right)$. Thus, $O_i^I\left(\omega'\right)$ and $O_i^L\left(\omega'\right)$ are common knowledge for i and b. We first prove Lemma 2 for the case that $O_i^L\left(\omega'\right) = 0$ by induction on $O_i^I\left(\omega'\right)$. We later extend the proof to include any $O_i^L\left(\omega'\right) > 0$.

Consider a link ib such that ib^G and $\left|S_b^{G(\omega')}\right| - \left|S_b^{G(\omega')-ib}\right| \geq m$. By Fact 1 this is common knowledge for i and b. If $O_i^L\left(\omega'\right) = 0$, $O_i^I\left(\omega'\right) = 0$, and $b \in \sigma_i\left(h_i\left(\omega'\right)\right)$ then b's unique best response is $i \in \sigma_b\left(h_b\left(\omega'\right)\right)$. Thus, i's unique best response is $b \in \sigma_i\left(h_i\left(\omega'\right)\right)$. Now, let $O_i^L\left(\omega'\right) = 0$ and let k be a non negative integer k such that for all $O_i^I\left(\omega'\right) \leq k$, i's unique best response is $b \in \sigma_i\left(h_i\left(\omega'\right)\right)$ and b's unique best response is $i \in \sigma_b\left(h_b\left(\omega'\right)\right)$. We now prove that this is also true for $O_i^L\left(\omega'\right) = 0$ and $O_i^I\left(\omega'\right) = k+1$. Note that since $\bar{L}_i^G \subseteq L_i^{G(\omega')}$ then $\left|S_b^{G(\omega')}\right| - \left|S_b^{G(\omega')-ib}\right| \geq m$ and this is the case as long as b does not default on i and ib is not eliminated. Thus, feasible network changes that are observable to i according to K_i^∞ include a disconnection of ib due to b's default on i, or a decrease in $O_i^I\left(\omega'\right)$ without b defaulting on i. Assume the latter, then by the induction assumption, in the new state ω'', i's unique best response is $b \in \sigma_i\left(h_i\left(\omega''\right)\right)$ and b's unique best response is $i \in \sigma_b\left(h_b\left(\omega''\right)\right)$. Therefore in state ω' (and in any state in which K_i^∞ is identical to $K_i^\infty\left(G\left(\omega'\right)\right)$) if $b \in \sigma_i\left(h_i\left(\omega'\right)\right)$ then b's unique best response is $i \in \sigma_b\left(h_b\left(\omega'\right)\right)$, hence i's unique best response is $b \in \sigma_i\left(h_i\left(\omega'\right)\right)$. This concludes the proof that for $O_i^L\left(\omega'\right) = 0$

and any $O_i^I(\omega')$, i's unique best response is $b \in \sigma_i(h_i(\omega'))$ and b's unique best response is $i \in \sigma_b(h_b(\omega'))$.

We now prove Lemma 2 for any $O_i^I(\omega')$ and $O_i^L(\omega')$. As a first step, assume that for some non negative integer k, if $O_i^L(\omega') \leq k$ then the claim is true for any $O_i^I(\omega')$. We now prove that if $O_i^L(\omega') = k+1$ then the claim is true for $O_i^I(\omega') = 0$. Note that since $\bar{L}_i^G \subseteq L_i^{\omega'}$ then $\left|S_b^{G(\omega')}\right| - \left|S_b^{G(\omega')-ib}\right| \geq m$ and this is the case as long as b does not default on i and ib is not eliminated. In addition, this is common knowledge to i and b. If $O_i^I(\omega') = 0$ and $O_i^L(\omega') > 0$ then feasible network changes that are observable to i according to K_i^∞ include a disconnection of ib due to b's default on i, or a decrease in $O_i^L(\omega')$ without b defaulting on i. Assume the latter, then the new state ω'' is such that $O_i^L(\omega'') = k$. So by the induction assumption, i's unique best response is $b \in \sigma_i(h_i(\omega''))$ and b's unique best response is $i \in \sigma_b(h_b(\omega''))$. Therefore, in state ω' (and in any state in which K_i^∞ is identical to $K_i^\infty(\omega')$) if $b \in \sigma_i(h_i(\omega'))$ then b's unique best response is $i \in \sigma_b(h_b(\omega'))$. Consequently, i's unique best response is $b \in \sigma_i(h_i(\omega'))$.

Finally, assume that for some non negative integers k_L and k_I the claim is true for any $O_i^L(\omega') \leq k_L$ regardless of $O_i^I(\omega')$, as well as for $O_i^L(\omega') = k_L + 1$ if $O_i^I(\omega') \leq k_I$. We now prove that it is true for $O_i^L(\omega') = k_L+1$ and $O_i^I(\omega') = k_I+1$. As before, since $\bar{L}_i^G \subseteq L_i^{G(\omega')}$ then $\left|S_b^{G(\omega')}\right| - \left|S_b^{G(\omega')-ib}\right| \geq m$ and this is the case as long as b does not default on i and ib is not eliminated. Thus, feasible network changes that are observable to i according to K_i^∞ include a disconnection of ib due to b's default on i, a decrease in $O_i^I(\omega')$ without b defaulting on i, and a decrease in $O_i^L(\omega')$ without b defaulting on i. For either case, we can employ our induction assumptions as above.■

Corollary 9. *Consider an economy with one borrower and let $m = 1$. Consider any network G, belief profile μ, lender ℓ, and state ω' such that $G^0 = G$ and $\ell b^{G(\omega')}$. Consider any strategy profile σ such that (σ, μ) is a pure strategy PBE-K^∞. Then σ satisfies the following. If ℓ is a 1-local monopoly in G then in state ω':*

[1] ℓ keeps her link to b $(b \in \sigma_\ell(h_i(\omega')))$; and

[2] b does not default directly on ℓ $(\ell \in \sigma_b(h_b(\omega')))$.

Proof of Theorem 1. **Part 1 - locally monopolistic ⇒ robust:** Given Lemmas 2 and 1, Corollary 9, and the definition of LEB it is sufficient to note that for any σ that satisfies the conditions (1) - (3) below, $\forall_{\ell b^G} \left|S_b^G\right| - \left|S_b^{G-\ell b}\right| \geq m$ and $\forall_{\ell i b^G} \left|S_b^G\right| - \left|S_b^{G-ib}\right| \geq m$ imply that G is sustained indefinitely with probability 1.

(1) $\forall_\ell \sigma_\ell(\omega^0(G)) \supseteq I_\ell^G$.
(2) For any intermediary i, network G, and state ω' such that $G^0 = G$, $\bar{L}_i^G \subseteq L_i^{G(\omega')}$, and $ib^{G(\omega')}$, if i is an m-local monopoly in G then $i \in \sigma_b(h_b(\omega'))$ and $b \in \sigma_i(h_i(\omega'))$.
(3) If $m = 1$, for any lender ℓ, network G, and state ω' such that $G^0 = G$ and $\ell b^{G(\omega')}$, if ℓ is a 1-local monopoly in G then $b \in \sigma_\ell(h_\ell(\omega'))$; and $\ell \in \sigma_b(h_b(\omega'))$.

Part 2 - robust ⇒ locally monopolistic: We prove that if $\forall_{\mu \in LEB(G)} \exists_\sigma (\sigma, \mu) \in \Sigma(G)$, then $\forall_{\ell i b^G} \left|S_b^G\right| - \left|S_b^{G-ib}\right| \geq m$ and $\forall_{\ell b^G} \left|S_b^G\right| - \left|S_b^{G-\ell b}\right| \geq m$.

Consider a network G such that $\exists_{\ell i b^G} \left|S_b^G\right| - \left|S_b^{G-ib}\right| < m$. We proceed by constructing a belief profile $\bar{\mu} \in LEB(G)$ such that $\forall_\sigma (\sigma, \bar{\mu}) \notin \Sigma(G)$ which provides the necessary contradiction.

Consider the following network G': Starting from the network G, for each intermediary in G add links to m distinct lenders that are not connected to any other intermediary or to b (add lenders to L if necessary). Note that G' is m-locally monopolistic. Let $\bar{\mu}$ be a belief profile as follows: for each agent $j \in V$ the belief $\bar{\mu}_j$ puts probability 1 on the network being G' minus the links that they observe that do not exist (i.e. links that belong $K_j^{\infty}\left(\omega\left(G'\right)\right)$ but not to $K_j^{\infty}(\omega(G))$). We have shown in the proof of Lemma 1 that $\forall_G \bar{\mu} \in LEB(G)$.

Consider first a strategy profile σ that dictates that if the initial network is G' then all of the lenders in G' keep all of their connections to intermediaries at $t=0$ $\left(\forall_\ell \sigma_\ell\left(\omega^0\left(G'\right)\right) \supseteq I_\ell^{G'}\right)$. Now consider $\ell i b^G$ such that $\left|S_b^G\right| - \left|S_b^{G-ib}\right| < m$, and assume by contradiction that $\left(\sigma, \bar{\mu}\right) \in \Sigma(G)$. Suppose that at some point in time, b defaults on i and denote by ω' the new state immediately after the elimination of ib. Given Lemma 2 and according to the beliefs of i and all of the lenders connected to i, $\forall_{i'b^G \neq ib} i' \in \sigma_b\left(h_b\left(\omega'\right)\right)$ and $b \in \sigma_{i'}\left(h_{i'}\left(\omega'\right)\right)$. Therefore, given $\bar{\mu}$ and for every $\ell i'^G \neq \ell i$, ℓ's unique best response is $i' \in \sigma_\ell\left(h_\ell\left(\omega'\right)\right)$. Now consider again the network G. Both i and b know that $\left|S_b^G\right| - \left|S_b^{G-ib}\right| < m$. Hence, according to $\bar{\mu}_i$ it must be the case that $i \notin \sigma_b\left(h_b\left(\omega^0(G)\right)\right)$. Consequently, $\left|S_b^G\right| - \left|S_b^{G-ib}\right| < m \Rightarrow b \notin \sigma_i\left(h_i\left(\omega^0(G)\right)\right)$.

Now consider a lender ℓ for which a connection to some intermediary i exists in G' and does not exist in G. Consider any σ such that ℓ disconnects her link to i at $t=0$ – remember that i is the (unique) intermediary that ℓ is connected to. Assume by contradiction that $\left(\sigma, \bar{\mu}\right) \in \Sigma(G)$. Since $\left(\sigma, \bar{\mu}\right) \in \Sigma(G)$ it must be that $\forall_{\omega \in \{\omega' | G^0 = G\} \cap \Omega(G), \ell' \in G} \sigma_{\ell'}(\omega) \supseteq I_{\ell'}^G$ and $\forall_{\omega \in \{\omega' | G^0 = G\} \cap \Omega(G), i' \in G} \sigma_{i'}(\omega) = B_{i'}^G$. Consider a strategy $\widehat{\sigma} = \left(\widehat{\sigma}_\ell, \sigma_{-\ell}\right)$ where $\widehat{\sigma}_\ell$ is identical to σ_ℓ with the only exception that $i \in \widehat{\sigma}_\ell\left(\omega^0\left(G'\right)\right)$. This changes only the information sets of b, i, and ℓ. Thus, the actions of all other agents are not affected as long as ω is such that $ib^{G(\omega)}$. We now show that given $\widehat{\sigma}$ and given that $\left(\sigma, \bar{\mu}\right) \in \Sigma(G)$, b's unique best response is not to defaults on i. Therefore, σ such that $i \notin \sigma_\ell\left(\omega^0(G)\right)$ is not an equilibrium strategy given $\sigma_{-\ell}$ and $\bar{\mu}$ – contradicting out assumption that $\left(\sigma, \bar{\mu}\right) \in \Sigma(G)$.

Suppose that given $\widehat{\sigma}$ there exists a sequence of actions that makes it strictly profitable for b to default on i, then the same sequence would have made it strictly profitable for b to default on i if ℓ was not connected to i. Moreover if given $\widehat{\sigma}$ there exists a sequence of actions that makes it weakly profitable for b to default on i, then the same sequence would have made it strictly profitable for b to default on i if ℓ was not connected to i. This is true because $i \in \widehat{\sigma}_\ell\left(\omega^0(G)\right)$ implies that $\ell i b^G$ as long as ib^G. Thus, if ℓ is connected to i, b losses strictly more from the elimination of the link ib.

We are left to prove that $\exists_{\ell b^G} \left|S_b^G\right| - \left|S_b^{G-\ell b}\right| < m$ implies that there exists a belief profile $\mu \in LEB(G)$ such that $\forall_\sigma \left(\sigma, \mu\right) \notin \Sigma(G)$. The proof involves an almost exact repetition of the argument above and is omitted.■

Corollary 10. *For any network G, there exists at least one ELEB. Moreover, there exists a belief profile μ such that μ is a ELEB for any network G* $\left(\exists_\mu \forall_G \mu \in LEB_E(G)\right)$.

The proof is identical to that of Lemma 1 and is omitted.

Let $\mu \in \mathcal{M}(G)$ if μ is consistent with the following condition for all G and $\ell i b^G$: for any borrower b, if $\ell i b^G \wedge \ell i b'^G$ then according to μ_j, with probability one $b = b'$. The following Corollaries are used in the proof of Theorem 2.

Corollary 11. *For any G*, $LEB(G) \cap \mathcal{M}(G) \neq \emptyset$. Moreover, $\exists_\mu \forall_G \mu \in LEB(G) \cap \mathcal{M}(G)$.

Corollary 12. *Consider any network G, belief profile* $\mu \in \mathcal{M}(G)$*, intermediary* i*, and state* ω' *such that* $G^0 = G$*,* $\bar{L}_i^G \subseteq L_i^{G(\omega')}$*, and* $\ell' i b^{G(\omega')}$ *for at least one lender* ℓ'*. Consider any strategy profile* σ *such that* (σ,μ) *is a pure strategy PBE-*K^∞ *and according to* σ *all lenders keep all of their links to intermediaries in G at time* 0 $\left(\forall_\ell \sigma_\ell\left(\omega^0(G)\right) \supseteq I_\ell^G\right)$*. Then* σ *satisfies the following. If* i *is an* m*-local monopoly in G then in state* ω'*:*
[1] i *keeps her link to* b $\left(b \in \sigma_i\left(h_i\left(\omega'\right)\right)\right)$*; and*
[2] b *does not default on* i $\left(i \in \sigma_b\left(h_b\left(\omega'\right)\right)\right)$.

Corollaries 11 and 12 follow directly from the proofs of Lemma 1 and Lemma 2 respectively.

Corollary 13. *Consider any network G, belief profile* $\mu \in LEB_E(G)$*, intermediary* i*, and state* ω' *such that* $G^0 = G$*,* $\bar{L}_i^G \subseteq L_i^{G(\omega')}$*, and* $\ell' i b^{G(\omega')}$ *for at least one lender* ℓ'*. Consider any invariant investment strategy profile* σ *such that* (σ,μ) *is a pure strategy PBE-*K^∞ *and according to* σ *all lenders keep all of their links to intermediaries in G at time* 0 $\left(\forall_\ell \sigma_\ell\left(\omega^0(G)\right) \supseteq I_\ell^G\right)$*. Then* σ *satisfies the following. If* i *is an* m*-local monopoly in G then in state* ω'*:*
[1] i *keeps her link to* b $\left(b \in \sigma_i\left(h_i\left(\omega'\right)\right)\right)$*; and*
[2] b *does not default on* i $\left(i \in \sigma_b\left(h_b\left(\omega'\right)\right)\right)$.

Proof of Corollary 13. By the definition of an invariant investment strategy, for any $j \in L \cup I$, $\sigma_j\left(\omega'\right) = \sigma_j(\omega) \cap \left(I \cup \widetilde{B}\right)$ for any state ω' in which [1] j can make a network adjustment; and [2] $K_j(G(\omega))|_{\widetilde{B}} = K_j\left(G\left(\omega'\right)\right)$ for some $\widetilde{B} \subseteq B$. In particular this is true for all $\widetilde{B} \in \left\{B' \subseteq B \middle| \left|B'\right| = 1\right\}$. Given that in the one borrower model $LEB_E(G) \equiv LEB(G)$, Lemma 2 applies directly to any state ω' satisfying [1] and [2].■

Corollary 14. *Let* $m = 1$*, and consider any network G, belief profile* μ*, lender* ℓ*, and state* ω' *such that* $G^0 = G$ *and* $\ell b^{G(\omega')}$*. Consider any invariant investment strategy profile* σ *such that* (σ,μ) *is a pure strategy PBE-*K^∞*. Then* σ *satisfies the following. If* ℓ *is a* 1*-local monopoly in G then in state* ω'*:*
[1] ℓ *keeps her link to* b $\left(b \in \sigma_\ell\left(h_i\left(\omega'\right)\right)\right)$*; and*
[2] b *does not default directly on* ℓ $\left(\ell \in \sigma_b\left(h_b\left(\omega'\right)\right)\right)$.

Proof of Theorem 2. **Part 1 - locally monopolistic ⇒ robust:** Given Corollaries 10, 13, and 14, and the definition of ELEB we are left only to note that given σ that satisfies the three conditions below, $\forall_{\ell b^G} \left|S_b^G\right| - \left|S_b^{G-\ell b}\right| \geq m$ and $\forall_{\ell i b^G} \left|S_b^G\right| - \left|S_b^{G-ib}\right| \geq m$ implies that G is sustained indefinitely with probability 1:

(1) $\forall_\ell \sigma_\ell\left(\omega^0(G)\right) \supseteq I_\ell^G$.
(2) For any intermediary i, network G, and state ω' such that $G^0 = G$, $\bar{L}_i^G \subseteq L_i^{G(\omega')}$, and $i b^{G(\omega')}$, if $\left|S_b^G\right| - \left|S_b^{G-ib}\right| \geq m$ then $i \in \sigma_b\left(h_b\left(\omega'\right)\right)$ and $b \in \sigma_i\left(h_i\left(\omega'\right)\right)$.
(3) If $m = 1$, then for any lender ℓ, network G, and state ω' such that $G^0 = G$ and $\ell b^{G(\omega')}$, if ℓ is a 1-local monopoly in G then $b \in \sigma_\ell\left(h_i\left(\omega'\right)\right)$; and $\ell \in \sigma_b\left(h_b\left(\omega'\right)\right)$.

Part 2 - robust ⇒ locally monopolistic: We prove that if $\forall_{\mu \in LEB_E(G)} \exists_\sigma (\sigma,\mu) \in \Sigma(G)$, then $\forall_{\ell i b^G} \left|S_b^G\right| - \left|S_b^{G-ib}\right| \geq m$ and $\forall_{\ell b^G} \left|S_b^G\right| - \left|S_b^{G-\ell b}\right| \geq m$.

Consider a network G such that $\exists_{\ell i b^G} \left|S_b^G\right| - \left|S_b^{G-ib}\right| < m$. We proceed by constructing a belief profile $\bar{\mu} \in LEB_E(G)$ such that $\forall_\sigma \left(\sigma, \bar{\mu}\right) \notin \Sigma(G)$ which provides the necessary contradiction. Consider the following network G': Starting from the network G, for each intermediary in G add links to m distinct lenders that are not connected to any other intermediary or borrower (add lenders to L if necessary). Note that G' is m-locally monopolistic. Let $\tilde{\mu}$ be a belief profile as follows: for each agent $j \in L \cup I$, the belief $\tilde{\mu}_j$ puts probability 1 on the network being the network G' minus the links that they observe that do not exist, and for each borrower b the belief $\tilde{\mu}_b$ puts probability 1 on the entire network being $K_b^\infty(G)$. By definition, $\tilde{\mu} \in \mathcal{M}(G)$ and from the proof of Lemma 1 and the definition of ELEB $\forall_G \tilde{\mu} \in LEB_E(G)$.

The reminder of the proof follows closely Part 2 of the proof of Theorem 1 noting that given belief $\tilde{\mu}$ the strategic considerations of any borrower b are identical to the strategic considerations in the case that b is the unique borrower in the economy and the beliefs are $\bar{\mu}$, and replacing belief $\bar{\mu}$ and Lemma 2 with the corresponding belief $\tilde{\mu}$ and Corollary 12.

We are left to prove that $\exists_{\ell b^G} \left|S_b^G\right| - \left|S_b^{G-\ell b}\right| < m$ implies that there exists a belief profile $\mu \in LEB_E(G)$ such that $\forall_\sigma \left(\sigma, \mu\right) \notin \Sigma(G)$. The proof involves an almost exact repetition of the argument above and is omitted.■

Proof of Proposition 2. Without loss of generality, we focus on the case where $m > 1$.
Part 1 - if: Consider an assessment $\left(\widehat{\sigma}, \widehat{\mu}\right)$ such that

(1) Belief profile $\widehat{\mu}$ is consistent with the following:

  (a) The belief of any borrower b puts probability 1 on the initial network being a network G' that includes exactly the following:

    (i) Links that exist with probability 1 according to b's knowledge $\left(K_b^\infty\left(G^0\right)\right)$;
    (ii) At least one additional borrower $b' \neq b$ such that for any subset L' of size m of the lenders that are financially related to b there exists an intermediary $i' \notin I_b^{G^0}$ who is connected only to borrower b' and to exactly all lenders in L'.
    (iii) The remainder of borrowers in the economy (if any) are not connected to any lender or intermediary.

  (b) The belief of any intermediary who is connected to strictly more or strictly less than m lenders in G^0 puts probability 1 on the initial network being G^0.
  (c) The belief of any intermediary i who is connected to exactly m lenders in G^0 puts probability 1 on the initial network being such that any borrower that i is connected to is financially related to exactly m lenders.
  (d) The belief of any lender ℓ puts probability 1 on the initial network being such that

    (i) There are only m lenders;
    (ii) Any borrower is financially related to exactly m lenders; and
    (iii) Any intermediary is connected to exactly m lenders.

(e) Any lender ℓ who observes a (zero probability event of) default or elimination of a link updates his belief to a belief that puts probability 1 on the following: any borrower in the economy is financially related to less than m lenders.

(2) Strategy profile $\widehat{\sigma}$ is consistent with the following:

(a) A borrower b defaults if and only if he is financially related to less than m lenders.
(b) At time $t=0$ an intermediary i keeps a link ib if and only if i's belief puts probability 1 on b being financially related to at least m lenders.
(c) Consider an intermediary i who is connected to exactly m lenders at time $t=0$. Then, i keeps a link ib unless [1] any lender disconnects from i; or [2] i observes an intermediary or lender disconnecting from b.
(d) At time $t=0$ a lender ℓ keeps all of her links.
(e) A lender ℓ disconnects from any intermediary i and borrower b if and only if he observes any link being eliminated.

It is left to show that:

(1) The assessment $\left(\widehat{\sigma},\widehat{\mu}\right)$ is a PBE-K^{∞}.
(2) Given $\left(\widehat{\sigma},\widehat{\mu}\right)$ a network $G^0=G$ is sustained indefinitely with probability 1 if each borrower has access to liquidity from at least m lenders or from none $\left(\forall_{b|\ell I b^G}\left|S_b^G\right|\geq m\right)$.

The latter is direct from the description of $\widehat{\sigma}$. Proving that $\left(\widehat{\sigma},\widehat{\mu}\right)$ is a PBE-K^{∞} requires: [i] showing that all of the actions ascribed by $\widehat{\sigma}$ are best responses given $\widehat{\mu}$ and $\widehat{\sigma}$; and [ii] verifying that $\widehat{\mu}$ does not contradict Bayes law. Both can be done by carefully going over the description above.

Part 2 - only if: This part is immediate, if a borrower can never lose m lenders, he always defaults.■

Proof of Proposition 3. For any network G, consider any belief profile μ^G that is consistent with the following:

(1) $\mu^G\in LEB_E(G)$.
(2) The belief of each intermediary i puts probability 1 on her being connected to all of the lenders who are financially related to any of the borrowers (i.e. no other lender is financially related to any borrower).
(3) If a lender ℓ is connected directly to a borrower b and if there is no intermediary i who is connected to both ℓ and b, then μ_ℓ^G puts probability 1 on b not being financially related to any lender besides ℓ.
(4) If a lender ℓ is connected to an intermediary i who is connected to a borrower b, then μ_ℓ^G puts probability 1 on i being connected to all of the lenders who are financially related to b.

One can verify that there exists at least one such belief profile, and that for any network G and belief profile μ^G, if G does not correspond with conditions [1] and [2] in Proposition 3 then there is no pure strategy profile σ' such that $\left(\sigma',\mu^G\right)\in\Sigma(G)$.■

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